Long-Term Care and Births Timing

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Two demographic trends

- Two demographic trends at work in the 21st century:

  1. The rise in the number of dependent elderly persons in need of LTC (linked to longevity growth)
  2. The postponement of births (since the 1970s)

Motivations: the LTC challenge

- The number of dependent elderly in EU-27 is expected to grow from 38 millions in 2010 to 57 millions in 2060.
- LTC provision is expected to remain largely informal.

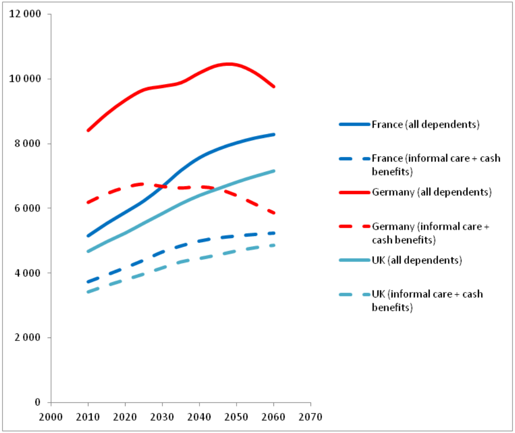


Figure: Number of dependent persons, in thousands (EU 2012)

Motivations: births postponement

- Because of various reasons (education, medical advances, earnings), individuals have children later on in their life (Gustafsson 2001).

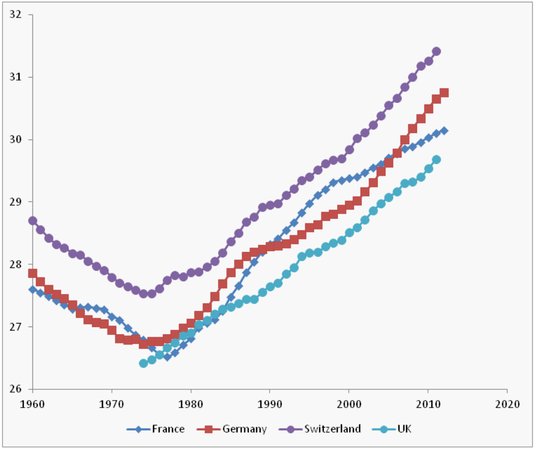


Figure: Mean age at birth (Human Fertility Database).

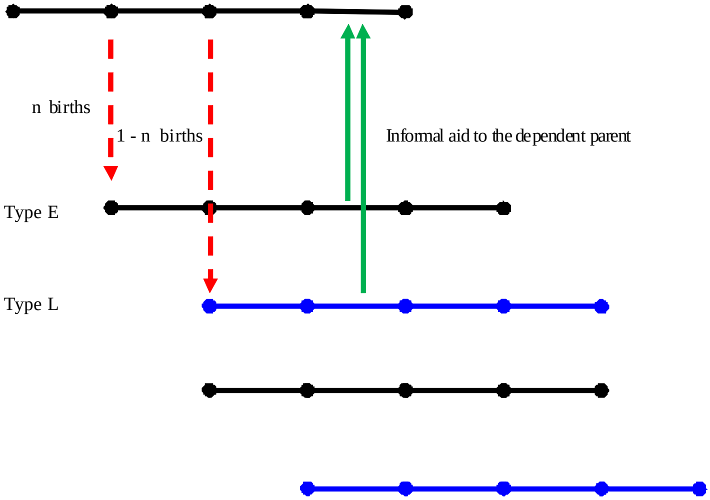
Two related phenomena

- Birth postponement raises the age gap between parents and children.
- The age gap determines the amount of LTC provided to parents.
- Fontaine et al (2007) using SHARE data:
  - The provision of informal LTC by children varies with the age of the child and his/her involvement on the labor market.
  - When younger children are still working full time, older children are more involved in the provision of LTC.

This paper

- We examine the conditions under which we can rationalize the stylized fact that early children provide more LTC.
- We study the design of optimal family policy.
- We develop a 4-period lifecycle fertility OLG model in order to study the joint decisions of birth timing and LTC provision.
  - no LTC insurance system (see LTC insurance puzzle Cremer et al 2012).
  - children provide informal LTC to their old dependent parents.
  - two types of agents ("early children" E and "late children" L) with different time constraints. Replacement fertility.

Lifecycle fertility in an OLG model



Our results

- We show that, at the laissez-faire, early children provide more LTC to their elderly parents in comparison to late children.
- In comparison to the social optimum, individuals tend, under weak conditions, to have too few early children and too many late children.
- The second-best uniform subsidy on early births depends on equity/efficiency concerns and on composition effects.

Related literature

- Family games and LTC
  - Konrad et al (2002), Wakabayashi and Horioka (2009), Pezzin et al (2007, 2009)
- Fertility as an insurance device for LTC
  - Cremer et al (2013)
- Optimal policy under LTC
  - Jousten et al (2005), Pestieau and Sato (2006, 2008), Cremer and Pestieau (2010), Cremer and Roeder (2012)
- Models of lifecycle fertility
  - d'Albis et al (2010), Pestieau and Ponthiere (2014, 2015)

Outline

The model: demography

- 4-period OLG model (each period has length 1):
  - period 1: childhood (no work);
  - period 2: work, consume, save and have $n < 1$ children;
  - period 3: work during $z < 1$, consume and have $1 - n < 1$ children;
  - period 4: old-age dependency: receive LTC from children.
- There exist two types of agents, depending on the age of their parent:
  - Type-E agents: children born from young parents ("early" children);
  - Type-L agents: children born from older parents ("late" children).
- q_t is the proportion of young adults of type E at time t.

The model: health production

- The health of the dependent elderly of type $i \in \{E, L\}$ at time t is a function of informal care received by children:

$$H_t^i \equiv H\left(b_t^i\right)$$

where $H'\left(\cdot\right) > 0$ and $H''\left(\cdot\right) < 0$ and where b_t^i is defined as:

$$b_t^i = n_{t-2}^i a_t^E + (1 - n_{t-2}^i) a_t^L$$

where a_t^E is the LTC provided by each of the n_{t-2}^i early children, and a_t^L is the LTC provided by each of the $(1 - n_{t-2}^i)$ late children.

- We assume perfect substitutability between the informal LTC of early and late children (basic skills).

The model: preferences

- Preferences of a young adult of type $i \in \{E, L\}$ are represented by:

$$u(c_t^i) + v\left(n_t^i\right) + u(d_{t+1}^i) + v(1 - n_t^i) + \varphi(a^i) + \gamma H(b_{t+2}^{ie})$$

where:

- c_t^i is consumption at the young age.
- d_{t+1}^i denotes consumption in third period.
- n_t^i is early fertility, $1 - n_t^i$ is late fertility.
- a^i is the informal LTC *given* to the parent. It is equal to a_{t+1}^E for type E and to a_t^L for type L.
- $\varphi(a^i)$ is the utility of helping the parent (a shortcut to have positive informal LTC).
- b_{t+2}^{ie} is the expected LTC *received* from children at the old age.
- $0 < \gamma < 1$ captures the degree of foresignthness.
- $u'(\cdot) > 0$, $u''(\cdot) < 0$, $v'(\cdot) > 0$, $v''(\cdot) < 0$, $\varphi'(\cdot) > 0$ and $\varphi''(\cdot) < 0$.

The model: budget constraints

- Types E and L differ on their age at which their parent is dependent.
- Type E are old when their parent needs LTC:

$$\begin{aligned} w_t \left(1 - \sigma n_t^E\right) &= c_t^E + s_t^E \\ w_{t+1}^e z(1 - \sigma(1 - n_t^E) - \mathbf{a}_{t+1}^E) + R_{t+1}^e s_t^E &= d_{t+1}^E \end{aligned}$$

- Type L are young when their parent needs LTC:

$$\begin{aligned} w_t (1 - \sigma n_t^L - \mathbf{a}_t^L) &= c_t^L + s_t^L \\ w_{t+1}^e z \left(1 - \sigma(1 - n_t^L)\right) + R_{t+1}^e s_t^L &= d_{t+1}^L \end{aligned}$$

where:

- σ is the time cost of children,
- s_t^E and s_t^L are savings,
- w_t is the hourly wage earned at time t,
- w_{t+1}^e is the expected wage rate at time $t+1$,
- R_{t+1}^e is equal to one plus the expected interest rate prevailing at $t+1$.

The model: production

- The production process involves capital K_t and labour L_t, and exhibits constant returns to scale:

$$Y_t = F(K_t, L_t)$$

- Full depreciation of capital K_t after one period of use.
- The labor force L_t is:

$$\begin{aligned} L_t &= q_t \left(1 - \sigma n_t^E\right) + (1 - q_t) \left(1 - \sigma n_t^L - a_t^L\right) \\ &\quad + q_{t-1} z (1 - a_t^E - \sigma(1 - n_{t-1}^E)) \\ &\quad + (1 - q_{t-1}) z \left(1 - \sigma(1 - n_{t-1}^L)\right) \end{aligned}$$

- Factors are paid at their marginal productivity:

$$\begin{aligned} w_t &= F_L \left(K_t, L_t\right) \\ R_t &= F_K \left(K_t, L_t\right) \end{aligned}$$

The laissez-faire: temporary equilibrium

Proposition

Given the anticipated future prices w^e_{t+1} and R^e_{t+1}, the anticipated future levels of LTC received a^{Ee}_{t+2} and a^{Le}_{t+2}, the capital stock K_t and the partitions q_{t-1} and q_t, the temporary equilibrium is a vector $\left\{c^E_t, d^E_{t+1}, n^E_t, a^E_{t+1}, c^L_t, d^L_{t+1}, n^L_t, a^L_t, w_t, L_t\right\}$ satisfying the conditions:

$$
\begin{aligned}
u'(c^i_t) &= R^e_{t+1} u'(d^i_{t+1}) \ \forall i \in \{E, L\} \\
u'(c^i_t)\sigma\left(w_t - \frac{w^e_{t+1}z}{R^e_{t+1}}\right) &= \left[\begin{array}{l} v'\left(n^i_t\right) - v'\left(1-n^i_t\right) \\ +\gamma H'(b^{ie}_{t+2})\left(a^{Ee}_{t+2} - a^{Le}_{t+2}\right)\end{array}\right] \forall i \in \{E, L\} \\
\varphi'(a^E_{t+1}) &= u'(c^E_t)\frac{w^e_{t+1}z}{R^e_{t+1}} \text{ and } \varphi'(a^L_t) = u'(c^L_t) w_t \\
w_t &= F_L\left(K_t, L_t\right) \\
L_t &= \left[\begin{array}{l} q_t\left(1-\sigma n^E_t\right) + (1-q_t)\left(1-\sigma n^L_t - a^L_t\right) \\ +q_{t-1}z(1-a^E_t - \sigma(1-n^E_{t-1})) \\ +(1-q_{t-1})z\left(1-\sigma(1-n^L_{t-1})\right)\end{array}\right]
\end{aligned}
$$

The laissez-faire: temporary equilibrium

Proposition

Under $\frac{w^e_{t+1} z}{R^e_{t+1}} < w_t$*, individuals of type E provide, in comparison with type-L individuals, a larger amount of LTC to their elderly parents, they consume also more and have more early children than type-L individuals:*

$$
\begin{aligned}
a^E_{t+1} &> a^L_t \\
c^E_t &> c^L_t \\
d^E_{t+1} &> d^L_{t+1} \\
n^E_t &> n^L_t
\end{aligned}
$$

- Under myopic anticipations, the condition vanishes to $z < R_t$ (weak).
- Under that condition, type L face a larger opportunity cost of providing LTC than type E.
- Impact of time constraints also on births timing.

The laissez-faire: stationary equilibrium

Proposition

The stationary equilibrium is a vector $\left\{c^E, d^E, n^E, a^E, b^E, c^L, d^L, n^L, a^L, b^L, K, L, w, R, q\right\}$ *satisfying:*

$$
\begin{aligned}
u'(c^i) &= Ru'(d^i)\ \forall i \in \{E, L\} \\
u'(c^i)w\sigma\left[1 - \frac{z}{R}\right] &= \left[\begin{array}{l} v'\left(n^i\right) - v'\left(1 - n^i\right) \\ +\gamma H'(b^i)\left(a^E - a^L\right)\end{array}\right] \forall i \in \{E, L\} \\
\varphi'(a^E) &= u'(c^E)\frac{wz}{R} \text{ and } \varphi'(a^L) = u'(c^L)w \\
K &= \left[\begin{array}{l} q\left(w\left(1 - \sigma n^E\right) - c^E\right) \\ +(1-q)\left(w\left(1 - \sigma n^L - a^L\right) - c^L\right)\end{array}\right] \\
L &= \left[\begin{array}{l} q\left(n^L\sigma(1-z) - n^E\sigma(1-z) + a^L - za^E\right) \\ +1 - \sigma n^L - a^L + z - \sigma z + \sigma z n^L\end{array}\right] \\
q &= \frac{n^L}{1 - n^E + n^L};\ w = F_L(K, L) \text{ and } R = F_K(K, L)
\end{aligned}
$$

The laissez-faire: stationary equilibrium

Proposition

At the stationary equilibrium, and assuming $R > z$, type-E agents provide more LTC to their parents, in comparison with type-L agents. They also have more early children, consume more and benefit from more LTC at the old age:

$$\begin{aligned} a^E &> a^L \text{ and } n^E > n^L \\ c^E &> c^L \text{ and } d^E > d^L \\ b^E &> b^L \end{aligned}$$

- Children of types E and L of the *same* parent provide unequal amounts of care, despite same preferences.

The laissez-faire: existence of stationary equilibrium

Proposition

Consider our economy with a log-linear utility function

$$(1-\delta)\log(c_t^i) + \delta \log\left(n_t^i\right) + (1-\delta)\log(d_{t+1}^i)$$
$$+\delta \log\left(1-n_t^i\right) + \eta \log(a^i) + \gamma \log(b_{t+2}^{ie})$$

and a Cobb-Douglas production function $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$.
Suppose $\gamma = 0$ *(full myopia) and* $\sigma = 0$ *(no time cost of children).*
Suppose $2(1-\delta) > \eta$ *and* $2(1-\delta)(1-\alpha)z + z\eta > \alpha\eta$.
Denote $\Gamma \equiv z\left[(1-\alpha)\left[2(1-\delta)+\eta\right] + \alpha\eta\right]$ *and*
$\Theta \equiv 2\eta\left[1-\delta\right]\left[\left[4(1-\delta)+\eta\right](1+z)\alpha + z(1-\alpha)\left[2(1-\delta)+\eta\right]\right]$.
If:

$$\Gamma\eta^2 + 4\left[1-\delta\right]^2 \alpha\eta < \Theta < \Gamma 4\left[1-\delta\right]^2 + \alpha\eta^3$$

then there exists at least one stationary equilibrium with perfect foresight such that $0 < 1 - a^i < 1 \forall i$.

The long-run social optimum: planning problem

- The utilitarian planner chooses consumptions, fertility and LTC to maximize social welfare in the stationary equilibrium.

- The problem of the social planner can be written by means of the following Lagrangian (where γ is set to 1: no myopia):

$$\max_{\substack{c^E,d^E,a^E,n^E\\ c^L,d^L,a^L,n^L,K}} \left[\begin{array}{l} \frac{n^L}{1-n^E+n^L}\left[\begin{array}{c} u(c^E)+v(n^E)+u(d^E)+v(1-n^E) \\ +H\left(n^E a^E+(1-n^E)a^L\right)+\varphi(a^E) \end{array}\right] \\ +\frac{1-n^E}{1-n^E+n^L}\left[\begin{array}{c} u(c^L)+v(n^L)+u(d^L)+v(1-n^L) \\ +H\left(n^L a^E+(1-n^L)a^L\right)+\varphi(a^L) \end{array}\right] \\ +\lambda F\left(K,\left[\begin{array}{c} \frac{n^L\left(n^L(\sigma-z\sigma)-n^E(\sigma-z\sigma)+a^L-za^E\right)}{1-n^E+n^L} \\ +1-\sigma n^L-a^L+z-\sigma z+\sigma z n^L \end{array}\right]\right) \\ +\lambda\left(-\frac{n^L}{1-n^E+n^L}\left(c^E+d^E\right)-\frac{1-n^E}{1-n^E+n^L}\left(c^L+d^L\right)-K\right) \end{array}\right]$$

where λ is the Lagrange multiplier.

The long-run social optimum: solution

Proposition

The long-run social optimum is a vector $\left\{c^{E*}, c^{L*}, d^{E*}, d^{L*}, a^{E*}, a^{L*}, b^{E*}, b^{L*}, n^{E*}, n^{L*}, K^{*}, L^{*}, q^{*}\right\}$ *such that:*

$$
\begin{aligned}
c^{E*} &= c^{L*} = d^{E*} = d^{L*} = c^{*} \\
n^{E*} &= n^{L*} = n^{*} \text{ and } b^{E*} = b^{L*} = b^{*} \\
\begin{bmatrix} v'(n^{*}) - v'(1-n^{*}) \\ +H'(b^{*})\left(a^{E*} - a^{L*}\right) \end{bmatrix} &= \begin{bmatrix} u'(c^{*})F_L(K^{*},\cdot) \\ \left[\sigma(1-z) - \left(a^{L*} - za^{E*}\right)\right] \\ -\left[\varphi(a^{E*}) - \varphi(a^{L*})\right] \end{bmatrix} \\
F_K(K^{*},\cdot) &= 1 \text{ and } q^{*} = n^{*} \\
\varphi'(a^{E*}) &= u'(c^{*})F_L(K^{*},\cdot)z - H'(b^{*}) \\
\varphi'(a^{L*}) &= u'(c^{*})F_L(K^{*},\cdot) - H'(b^{*}) \\
&\Longrightarrow a^{E*} > a^{L*} \\
L^{*} &= \begin{bmatrix} q^{*}\left(a^{L*} - za^{E*}\right) + 1 - \sigma n^{*} \\ -a^{L*} + z - \sigma z + \sigma z n^{*} \end{bmatrix}
\end{aligned}
$$

The long-run social optimum versus the laissez-faire

Proposition

Comparing the laissez-faire (i) under $R > z$ with the social optimum (i∗):

- $K^i \lesseqgtr K^{i*}$ *when* $R \gtrless 1$ *prevails at the laissez-faire.*
- $c^{i*} = d^{i*}$, *whereas* $c^i \lesseqgtr d^i$ *when* $R \gtrless 1$ *prevails at the laissez-faire.*
- $a^{E*} > a^E$ *and* $a^{L*} > a^L$ *if*

$$\begin{aligned} u'(c^E)F_L(K,\cdot)\frac{z}{R} &> u'(c^*)F_L(K^*,\cdot)z - H'(b^*) \\ u'(c^L)F_L(K,\cdot) &> u'(c^*)F_L(K^*,\cdot) - H'(b^*) \end{aligned}$$

- $n^* > n^E > n^L$ *if*

$$\begin{bmatrix} u'(c^E)F_L(K,\cdot)\sigma \\ -\gamma H'(b^E)(a^E - a^L) \end{bmatrix} > \begin{bmatrix} u'(c^*)F_L(K^*,\cdot)(\sigma - a^{L*} + za^{E*}) \\ -[\varphi(a^{E*}) - \varphi(a^{L*})] \\ -H'(b^*)(a^{E*} - a^{L*} + \sigma) \end{bmatrix}$$

- $b^* > b^E > b^L$ *under those conditions.*

The long-run social optimum: decentralization

Proposition

The long-run social optimum can be decentralized by means of:

- *Intergenerational lump-sum transfers allowing K to reach K^*.*
- *Intra-generational lump-sum transfers equalizing c across types.*
- *Subsidies on early births θ^E and θ^L equal to:*

$$\theta^{i*} = F_L(K^*, \cdot)\left[\left(a^{L*} - za^{E*}\right)\right] + \frac{\varphi(a^{E*}) - \varphi(a^{L*})}{u'(c^*)} + \frac{(a^{E*} - a^{L*})\begin{bmatrix} H'\left(n^* a^{E*} + (1-n^*)a^{L*}\right) \\ -\gamma H'\left(n^i a^{E*} + (1-n^i)a^{L*}\right) \end{bmatrix}}{u'(c^*)}$$

- *Subsidies on LTC to the elderly parents equal to:*

$$\mu^{E*} = \mu^{L*} = \frac{H'\left(n^* a^{E*} + (1-n^*)a^{L*}\right)}{u'(c^*)}$$

The second-best problem

- The decentralization of the first-best requires policy instruments that are hardly available.
- Here we consider only three instruments:
  - a tax on labor earnings τ
  - a demogrant T
  - a uniform subsidy on early children θ.
- Simplifying assumptions:
  - the cost of children is here defined in terms of goods
  - a small open economy at the stationary equilibrium (w is fixed and $R = 1$)
  - full myopia ($\gamma = 0$).

The second-best problem

Type E's decisions satisfy:

$$\begin{aligned} u'(c^E) &= u'(d^E) \\ u'(c^E)\sigma(1-\theta) &= v'(n^E) - v'(1-n^E) \\ u'(d^E)zw(1-\tau) &= \varphi'(a^E) \end{aligned}$$

Type L's decisions satisfy:

$$\begin{aligned} u'(c^L) &= u'(d^L) \\ u'(c^L)\sigma(1-\theta) &= v'(n^L) - v'(1-n^L) \\ u'(c^L)w(1-\tau) &= \varphi'(a^L) \end{aligned}$$

From these, we obtain the following demand functions:

$$\begin{aligned} s^i &= s^i(\tau, \theta, T) \\ n^i &= n^i(\tau, \theta, T) \\ a^i &= a^i(\tau, \theta, T) \end{aligned}$$

The second-best problem

- The second-best planning problem can be written as the following Lagrangian $\mathcal{L}$:

$$
q\left[\begin{array}{l} u\left(w(1-\tau)-s^E-\sigma n^E(1-\theta)+T\right) \\ +u\left(wz(1-\tau)(1-a^E)+s^E-\sigma(1-n^E)\right) \\ +v(n^E)+v(1-n^E)+\varphi(a^E)+H(\hat{b}^E) \end{array}\right]
$$

$$
+(1-q)\left[\begin{array}{l} \left[u\left(w(1-\tau)(1-a^L)-s^L-\sigma n^L(1-\theta)+T\right)\right. \\ +u\left(wz(1-\tau)+s^L-\sigma(1-n^L)\right) \\ \left.+v(n^L)+v(1-n^L)+\varphi(a^L)+H(\hat{b}^L)\right] \end{array}\right]
$$

$$
+\mu\left[\begin{array}{l} \tau\left(q\left(w+(1-a^E)zw\right)+(1-q)(w(1-a^L)+zw)\right. \\ -\theta\sigma\left(qn^E+(1-q)n^L\right)-T \end{array}\right]
$$

where $\hat{b}^i = a^E n^i + a^L(1-n^i)$ and $q = \frac{n^L}{1+n^L-n^E}$.

The second-best problem

- Using the laissez-faire FOCs and the envelope theorem, we obtain:

$$\frac{\partial \mathcal{L}}{\partial s^i} = 0$$

$$\frac{\partial \mathcal{L}}{\partial a^E} = qH'(\hat{b}^E)n^E + (1-q)\,H'(\hat{b}^L)n^L - \mu\tau qzw$$

$$\frac{\partial \mathcal{L}}{\partial a^L} = (1-q)\,H'(\hat{b}^L)\left(1-n^L\right) + qH'(\hat{b}^E)(1-n^E) - \mu\tau(1-q)w$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n^E} &= \left[U^E - U^L + \mu\tau w\left(a^L - za^E\right)\right]\frac{\partial q}{\partial n^E} \\ &\quad + qH'(\hat{b}^E)(a^E - a^L) - \mu\theta\sigma\left[q + (n^E - n^L)\frac{\partial q}{\partial n^E}\right]\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n^L} &= \left[U^E - U^L + \mu\tau w\left(a^L + za^E\right)\right]\frac{\partial q}{\partial n^L} \\ &\quad + (1-q)\,H'(\hat{b}^L)(a^E - a^L) - \mu\theta\sigma\left[1 - q + (n^E - n^L)\frac{\partial q}{\partial n^L}\right]\end{aligned}$$

The second-best problem: earning tax

- If cross derivatives in compensated terms are negligible, the derivative of the compensated lagrangian is:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \tau} = \frac{\partial \mathcal{L}}{\partial \tau} + \frac{\partial \mathcal{L}}{\partial T}\bar{y} = -cov(u', y) + A - \mu \tau w \left[qz \frac{\partial \tilde{a}^E}{\partial \tau} + (1-q)\frac{\partial \tilde{a}^L}{\partial \tau} \right]$$

  where:

  -
    - $\bar{y} = w\left(q + (1-q)\left(1-a^L\right)\right) + wz\left(q(1-a^E) + (1-q)\right)$
    - $Eu' = q\left[u'(c^E) + u'(d^E)\right] + (1-q)\left[u'(c^L) + u'(d^L)\right]$
    - $Eu'y = q\left[u'(c^E)w + u'(d^E)zw(1-a^E)\right] + (1-q)\left[u'(c^L)w(1-a^L) + u'(d^L)zw\right]$
    - $A \equiv \frac{\partial \tilde{a}^E}{\partial \tau}\left[qH'(\hat{b}^E)n^E + (1-q)H'(\hat{b}^L)n^L\right] + \frac{\partial \tilde{a}^L}{\partial \tau}\left[qH'(\hat{b}^E)(1-n^E) + (1-q)H'(\hat{b}^L)(1-n^L)\right]$
    - $\frac{\partial \tilde{a}^i}{\partial \tau} \equiv \frac{\partial a^i}{\partial \tau} + \frac{\partial a^i}{\partial T}\frac{\partial T}{\partial \tau} = \frac{\partial a^i}{\partial \tau} + \frac{\partial a^i}{\partial T}\bar{y}.$

- Equalizing $\frac{\partial \tilde{\mathcal{L}}}{\partial \tau}$ to 0 and isolating τ yields...

The second-best problem: earning tax

Solution (optimal earning tax)

$$\tau = \frac{-cov(u', y) + A}{\mu w \left[qz \frac{\partial \tilde{a}^E}{\partial \tau} + (1-q) \frac{\partial \tilde{a}^L}{\partial \tau} \right]}$$

- The covariance term is negative, and captures equity concerns
- A captures the incidence of earnings tax on the provision of LTC by children

$$A \equiv \left[\begin{array}{l} \frac{\partial \tilde{a}^E}{\partial \tau} \left[qH'(\hat{b}^E) n^E + (1-q) H'(\hat{b}^L) n^L \right] \\ + \frac{\partial \tilde{a}^L}{\partial \tau} \left[qH'(\hat{b}^E)(1-n^E) + (1-q) H'(\hat{b}^L)(1-n^L) \right] \end{array} \right]$$

- The denominator is an efficiency term, which captures the incidence on the tax base

The second-best problem: family allowances

- Assuming that cross derivatives in compensated terms are negligible, the derivative of the compensated lagrangian:

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \theta} = \frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial \mathcal{L}}{\partial T}\bar{n}_E = Eu'_E n_E - \bar{n}_E Eu'_E + B + C - \theta D$$

  where:

  -
    - $\bar{n}_E \equiv \sigma\left(qn^E + (1-q)n^L\right)$ and $Eu'_E \equiv qu'\left(c^E\right) + (1-q)u'(c^L)$
    - $Eu'_E n_E \equiv \sigma\left[qn^E u'(c^E) + (1-q)n^L u'(c^L)\right]$
    - $B \equiv \left[U^E - U^L + \mu\tau(wa^L - zwa^E)\right]\left[\frac{\partial q}{\partial n^E}\frac{\partial \tilde{n}^E}{\partial \theta} + \frac{\partial q}{\partial n^L}\frac{\partial \tilde{n}^L}{\partial \theta}\right]$
    - $C \equiv (a^E - a^L)\left[qH'(\hat{b}^E)\frac{\partial \tilde{n}^E}{\partial \theta} + (1-q)\,H'(\hat{b}^L)\frac{\partial \tilde{n}^L}{\partial \theta}\right]$
    - $D \equiv \mu\sigma\left[\frac{\partial \tilde{n}^E}{\partial \theta}\left(q + \left(n^E - n^L\frac{\partial q}{\partial n^E}\right)\right) + \frac{\partial \tilde{n}^L}{\partial \theta}\left(1 - q + \left(n^E - n^L\frac{\partial q}{\partial n^L}\right)\right)\right]$
    - $\frac{\partial \tilde{n}^E}{\partial \theta} \equiv \frac{\partial n^E}{\partial \theta} + \frac{\partial n^E}{\partial T}\frac{\partial T}{\partial \theta} = \frac{\partial n^E}{\partial \theta} + \frac{\partial n^E}{\partial T}\bar{n}_E$
    - $\frac{\partial \tilde{n}^L}{\partial \theta} \equiv \frac{\partial n^L}{\partial \theta} + \frac{\partial n^L}{\partial T}\frac{\partial T}{\partial \theta} = \frac{\partial n^L}{\partial \theta} + \frac{\partial n^L}{\partial T}\bar{n}_E$

- Equalizing $\frac{\partial \tilde{\mathcal{L}}}{\partial \theta}$ to 0 and isolating θ yields...

The second-best problem: family allowances

Solution (optimal family allowance)

$$\theta = \frac{cov(u'_E, n_E) + B + C}{\mu\sigma \left[\frac{\partial \tilde{n}^E}{\partial \theta} \left(q + \left(n^E - n^L \frac{\partial q}{\partial n^E} \right) \right) + \frac{\partial \tilde{n}^L}{\partial \theta} \left(1 - q + \left(n^E - n^L \frac{\partial q}{\partial n^L} \right) \right) \right]}$$

- The covariance term is an equity term.
- B is the effect of composition on overall utility and earning tax revenue

$$B \equiv \left[U^E - U^L + \mu\tau(wa^L - zwa^E) \right] \left[\frac{\partial q}{\partial n^E} \frac{\partial \tilde{n}^E}{\partial \theta} + \frac{\partial q}{\partial n^L} \frac{\partial \tilde{n}^L}{\partial \theta} \right]$$

- C reflects the incidence of θ on the LTC provision by children

$$C \equiv (a^E - a^L) \left[qH'(\hat{b}^E) \frac{\partial \tilde{n}^E}{\partial \theta} + (1-q)\, H'(\hat{b}^L) \frac{\partial \tilde{n}^L}{\partial \theta} \right]$$

- The denominator is a standard efficiency term

Conclusions

- The timing of birth matters for LTC provision:
  - early children are older when their parents are dependent, and thus face a lower opportunity cost of LTC provision.
- From a policy perspective, early births should be encouraged, since these allow the society to benefit from cheaper LTC provision.
- In reality, there exist other reasons why the decentralized birth timing may not be socially optimal (education externalities etc).