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THE POLITICAL ECONOMY OF (IN)FORMAL LONG TERM CARE TRANSFERS

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The political economy of (in)formal long term care transfers[1]

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Abstract

We develop a model where families consist of one parent and one child, with children differing in income and all agents having the same probability of becoming dependent when old. Young and old individuals vote over the size of a social long term care transfer program, which children complement with informal (time) or formal (money) help to their dependent parent. Dependent parents have an intrinsic preference over informal to monetary help.
We first show that low (resp., high) income children provide informal (resp. formal) help, whose amount is decreasing (resp. increasing) with the child's income. The middle income class may give no family help at all, and its elderly members would be the main beneficiaries of the introduction of social LTC transfers. We then provide several reasons for the stylized fact that there are little social LTC transfers in most countries. First, social transfers are dominated by informal help when the intrinsic preference of dependent parents for informal help is large enough. Second, when the probability of becoming dependent is lower than one third, the children of autonomous parents are numerous enough to oppose democratically the introduction of social LTC transfers. Third, even when none of the first two conditions is satisfied, the majority voting equilibrium may entail no social transfers, especially if the probability of becoming dependent when old is not far above one third. This equilibrium may be local (meaning that it would be defeated by the introduction of a sufficiently large social program). This local majority equilibrium may be empirically relevant whenever new programs have to be introduced at a low scale before being eventually ramped up.

Keywords: Majority Voting, local Condorcet winner, crowding out, intrinsic preference for informal help, tax reform.

JEL codes: H55, I13, D91.

1 Introduction

Long Term Care (LTC hereafter) is defined as the care needed by people who are unable to perform alone activities of daily living (such as bathing, dressing, eating, getting in and out of bed, toileting and continence) or instrumental activities of daily living (preparing own meals, cleaning, laundry, taking medication, getting to places beyond walking distance, shopping, managing money affairs and using the telephone/Internet). People in need of LTC are called dependent.

The loss of autonomy is most often associated with old age and should be clearly distinguished from illness, disability and handicap. Financing and providing LTC is an important and growing problem for all rich countries. Torjman (2013) reports that the chances of requiring LTC in Canada are one in ten by age 55, three in ten by age 65 and five in ten by age 75 (Canadian Life and Health Insurance Association, 2012). Although Canada's population will not age quite as rapidly as that in many other advanced countries, its LTC needs will nevertheless increase dramatically. In 2036, 10 to 11 million Canadians will be aged 65 and older — more than double the 2009 figure of 4.7 million — and their numbers will continue to rise to some 12 to 15 million by 2061. As a consequence, the number of dependent seniors is expected to triple over the next 50 years (Grignon and Bernier, 2012 and Statistics Canada, 2010).

A major difference with health care is the importance of help from the family in coping with dependency. This help can take two forms: informal (help in time) or formal (monetary transfers). Most seniors with LTC needs reside in their or their relatives' home, and rely largely on volunteer care from family members. These include seniors with severe impairments (unable to perform at least four activities of daily living). In addition, many people who do pay for care in their home also rely on some free services. The economic value of volunteer care is significant, although estimates of it are highly uncertain (see Cremer *et al.*, 2009).

The formal LTC sector is relatively small (at an average of 1.5% of GDP across 25 OECD countries in 2008), especially when compared to the estimated value of family care and expenditure on health or pension systems (OECD, 2011). Public LTC programs provide either cash benefits or in-kind services, which depend on the needs and on the financial resources of the user.[1] Indeed, one reason given to the relative lack of public LTC spending (and also, the lack of private insurance) is precisely the importance of help from the family.

[1] OECD (2011, chapter 7) provides a detailed taxonomy of public LTC systems within the OECD. For broader surveys on long term care, see Brown and Finkelstein (2011) and Cremer et al. (2009).

We develop a model where agents choose the extent of both public LTC transfers (by majority voting) and of formal (money) and informal (time) help they provide to dependent members of their family. Our objective is twofold. First, recent empirical evidence (detailed below) has emerged on the pattern of formal and informal family help, and we want to shed light on these empirical regularities. For instance, we want to understand how the type of family help given is affected by the income level of the help giver, as well as the crowding out effects (if any) between public LTC transfers and the two forms of family help. Second, and more importantly, we want to better understand how it is that so little social LTC transfers are provided in reality.

We model a continuum of families, consisting of an old member (parent) and a young one (child). Young individuals work and obtain labor income. Old agents are retired. The only *ex ante* difference between families is the productivity of the child. All agents face the same probability π of becoming dependent when old. Children are altruistic towards their dependent parent, and can help them in three different ways. First, they can provide informal help, namely time. Second, they can provide formal help, in the form of a direct monetary transfer. Finally, they can support a social LTC transfer scheme, financed by a proportional tax on labor income, whose proceeds serve a lump sum benefit to all dependent agents. We assume away labor income tax distortions, in order to study the case most favorable to the introduction of a social LTC program. A very important assumption is that, other things equal, parents prefer to benefit from one hour of help by their child rather than by a professional. This assumption is in line with casual observations and is supported by Pinquart and Sorensen (2002).[2] We assume that the strength of this preference is identical for all dependent agents.

The timing of decisions runs as follows. At each period, all agents (young and old) vote over the proportional tax rate financing the social LTC transfers. Young agents then observe the majority-chosen level of this tax rate, and complement with formal and/or informal family help. The dependency status of the elderly in the family is already known when voting, so that our approach models LTC transfers rather than insurance.[3]

[2]It is also associated with the desire of most people to remain in their own homes for as long as possible. Recent polling by the Canadian Life and Health Insurance Association, reported in Frank (2012) found that 77 percent of Canadians would prefer to stay in their homes as they age. See also Torjman (2013).

[3]The same assumption is made by Nuscheler and Roeder (2013), detailed in section 1.1. Costa Font *et al.* (2014) contrast ex ante financing of LTC (such as insurance) with ex post financing (such as transfers, as in our model). They find that most OECD countries' LTC spending is financed by close to a fifty-fifty mix of ex-ante and ex-post funding sources or that the spending relies heavily on ex-post funding sources. They also obtain that OECD countries view the two forms of funding as substitutes rather than complements. We leave for future research the explanation of this very interesting observation.

We solve the model by backward induction and obtain the following results. First, low productivity children give informal help to their dependent parent, while high productivity children prefer giving formal help. The threshold child productivity determining the preferred type of family help is driven by the preference of the dependent for informal help. Informal help (among low productivity children) decreases with productivity (because of the higher opportunity cost of giving time) while formal help increases with productivity (thanks to a lower marginal utility of consumption of richer children). These comparative statics results are reminiscent of those obtained by Pestieau and Sato (2006, 2008) who use a different model (see section 1.1). Unlike them, we do not restrict ourselves to interior solutions for help of any type, and we obtain that the "middle class" (i.e., families where the child's productivity is neither low nor high) may not give any (in)formal family transfers to dependent agents. This result will play a very important role in the determination of the majority chosen value of the tax rate. We also note that the pattern of family help Pestieau and Sato (2006, 2008) and us obtain conforms to the empirical evidence. Zissimopoulos (2001) shows that the adult child transfers fewer hours of time and a greater amount of money to his dependent parent as his wage rate increases. Couch *et al.* (1999) obtain that time transfers respond negatively to wage rates of unmarried men and women, and of married men, while money transfers respond positively to wage rates of married and unmarried men and women. Sloan *et al.* (2002) find a positive and significant relationship between money transfers to the parents and hourly wage of the child.

We then turn to the main results of the paper, namely why there are so little social LTC transfers in OECD countries. We first show that these transfers crowd out (in)formal family help. If the intrinsic preference of dependent agents for informal help is large enough, then nobody most-prefers social transfers: poor children prefer to resort to informal help, while rich children prefer formal family help. To the best of our knowledge, we are the first to model explicitly the relationship between preferences of the dependent for informal help and the lack of social LTC transfers.

In the rest of the paper, we assume that the intrinsic preference for informal help is low enough that poor children actually prefer social LTC transfers to informal help (at their most-preferred allocation), a necessary condition for the emergence of a social LTC transfer program at the majority voting equilibrium. We show that the availability of social transfers, compared to the situation where only family help is available, benefits dependent parents both at the intensive and extensive margins (with a shrinking of the set of productivity levels for which dependent parents receive no help at all). In that sense, the introduction of social transfers benefits especially the "middle-income class" which is reminiscent of Director's law (Stigler, 1970). Moving then to the majority voting equilibrium, we first note that the probability of becoming dependent must be large enough (above one third) for a positive amount of social LTC transfers to be selected. This assumption looks empirically realistic (see the first paragraphs of this Introduction for instance, as well as Brown and Finkelstein, 2009). The main technical difficulty we then face is that preferences of dependent parents are not always single-peaked in the social contribution rate. This is due to the crowding out effect of social contributions on (in)formal help: the utility of some dependent parents first decreases with the tax rate (because a higher tax rate decreases the help they receive from their family) and then increases with it (when family help is totally crowded out, so that the only help received is the social transfer). Those agents then have V-shaped preferences over the tax rate. We identify a condition on crowding out of formal help by social transfers which guarantees that the preferences of all agents (young and old) satisfy the single-crossing property (Gans and Smart, 1996 and Epple and Romano, 1996).

We provide a joint condition on the probability of becoming dependent and on the distribution of productivities under which the majority voting equilibrium tax rate is nil. This condition is likely to be satisfied if the probability of becoming dependent is close enough to one third. We also look for a weaker equilibrium concept (a local Condorcet winner) where a majority of voters prefers this tax rate to any other value in a neighborhood. We provide a condition under which the local Condorcet winning value of the tax rate is nil. This raises an interesting possibility for why there are so little social LTC transfers: a majority may dislike small social LTC programs (because they crowd out part of family help) while actually preferring much larger programs, which totally crowd out family help but generate large social transfers. If policy changes are mainly incremental, so that new programs are small and then grow other time (which is the case of most social programs introduced since the beginning of the 20^{th} century), a local Condorcet winning value of the tax rate of zero may prevent the emergence of a social LTC transfer program.

Finally, we identify the possibility of a positive tax rate at equilibrium, supported by poor members of families with a dependent elderly. In that case, the equilibrium allocation is such that children of dependent parents whose income is sufficiently different from the decisive voter's income complement the social transfers with informal (for low income levels) or formal (for high income levels) help, while intermediate income levels (close to the decisive voter's income) rely exclusively on social transfers.

We now turn to the related literature.

1.1 Literature

There is a growing empirical literature on family help and its substitutability with formal LTC care. We mention this literature either above, or when we introduce a specific assumption in the model (see after Assumption 1, and Lemma 1). We then concentrate here on theoretical papers.

The papers closest to ours in their setting are Pestieau and Sato (2008) and especially Pestieau and Sato (2006). They both model families differing in the productivity of the child and whether the parent is dependent or not, as in our model. Their approach differs from ours on several counts. First, they model the advent of dependency as a utility loss, while all types of help bring utility gains, with the utility function for consumption unaffected. We believe our formulation, using state-dependent preferences, to be more in line with the recent empirical literature, such as Finkelstein *et al.* (2013). Second, they model a first stage decision where parents donate gifts to their children as a form of LTC insurance. Such a stage is not included in our model.[4] Third, and more important, they take a normative approach while ours is positive. They study various tax and transfer policies and conclude that the optimal policy "depends on its effect on parental gifts, on children's labor supply, on the distribution of wages and on consumption inequality between parents and children and between children having dependent parents and children having healthy parents." Pestieau and Sato (2008) build on the same model and introduce public and private nursing facilities, as well as private insurance.

Other papers also adopt a normative viewpoint and study the optimal public LTC policies in the presence of family help. Jousten *et al.* (2005) focus on families with different levels of altruism. Given the cost of public funds, the central planner tries to induce the more altruistic families to assist their dependent parents and to restrict aid to the dependent elderly whose children are less altruistic. This may imply a quantity of public LTC lower than its first-best level. Pestieau and Sato (2009) consider a society segmented into altruistic and non altruistic

[4] Sloan et al. (1997) find little empirical support for the hypothesis that care giving by children is motivated by the prospect of receiving bequests from their parents.

families and also into poor and rich families. Private insurance is available. In a world of perfect information, a redistributive government helps the low income families. In altruistic families, the dependent parents are taken care of by their children. In non altruistic families, if needed, parents are given the means to purchase private LTC insurance. They then introduce asymmetric information and show that it implies less redistribution towards the two target populations: the poor elderly with non altruistic children and the poor altruistic families. Cremer and Roeder (2013) study a setting where children are purely selfish, and neither side can make credible commitments (which rules out efficient bargaining). They show that a positive level of informal help is provided as long as the bequest motive is operative. They study how the provision of LTC can be improved by public policies under various informational assumptions. Interestingly, crowding out of private aid by public LTC is not a problem in their setting. Public insurance may even enhance the provision of informal aid under certain circumstances.

There are fewer papers taking a positive approach to understanding LTC public policies. De Donder and Leroux (2013) study whether behavioral biases may explain the lack of social LTC insurance. They develop a model where individuals vote over the social LTC insurance contribution rate before buying additional private insurance and saving. They study three types of behavioral biases, all having in common that agents under-weight their dependency probability when taking private decisions. Sophisticated procrastinators anticipate their mistake when voting, while optimistic and myopic agents have preferences that are consistent across choices. Optimists under-estimate their own probability of becoming dependent but know the average probability while myopics underestimate both. Sophisticated procrastinators attain the first-best allocation while myopics and optimists insure too little and save too much. Myopics and optimists more (resp., less) biased than the median are worse off (resp., better off), at the majority voting equilibrium, when private insurance is available than when it is not.

De Donder and Pestieau (2015) study the political determination of the level of social LTC insurance when voters can top up with private insurance, saving and formal family help. Agents differ in income, probability of becoming dependent and of receiving family help, and amount of family help received. Social insurance redistributes across income and risk levels, while private insurance is actuarially fair. The income-to-dependency probability ratio of agents determines whether they prefer social or private insurance. Family support crowds out the demand for both social and, especially, private insurance, as strong prospects of family help drive the demand for private insurance to zero. The availability of private insurance decreases the demand for social insurance but need not decrease its majority chosen level. A majority of voters would oppose banning private insurance.

Nuscheler and Roeder (2013)'s aim is to explain income redistribution and public financing of LTC, in a setting where voters differ in income and in LTC needs and where a proportional income tax finances both a lump-sum and a LTC transfer. They have four groups of agents (two income levels, dependent and autonomous) and assume that the fraction of dependent parents is larger than one half. They show that a structure-induced equilibrium always exists, that it is unique if informal care is provided in equilibrium, and that it is consistent with the negative correlation observed between income inequality and LTC public financing.

We now present our modelling assumptions, before solving for the individual decisions and then deriving the majority voting equilibrium.

2 The model

The economy is composed of a continuum of agents who live two periods of length $T > 1$. All agents have a unique child at the end of the first period, so that at any point in time there is a continuum of families composed of one parent and one child. All agents have the same probability π of becoming dependent at the beginning of their second period. By the law of large numbers, there is then a fraction π of families with a dependent elderly at any point in time.

Children are altruistic towards their parent when the latter is dependent.[5] They can help their parent through three channels: by giving informal help e, by making a direct financial transfer f (i.e., formal help) and through a social LTC transfer system serving a lump sum amount b to all dependent agents. All decisions affect the current period only. More precisely, we assume that the future, second period utility of an individual currently young is not affected by his decisions when young, so we can concentrate on his instantaneous utility.[6]

[5] We are interested in how much (more) young agents help their parent in case of dependency, so that any help to a non-dependent parent can be subsumed in the existing model. Note that we also assume away altruism from parents to children to focus on help by children to parents. See section 4 of Cremer et al. (2009) for a survey of strategic considerations related to family solidarity, including strategic bequests.

[6] Pestieau and Sato (2006, 2008) make the same assumption, although it remains implicit in their setting. This assumption requires that we assume away saving, strategic interactions between parents and children (such as exchanges of help for bequests) and that the size of the public LTC program is chosen at each period. See footnote 4 for the lack of strategic motivations for family help, and footnote 12 for a defense of the last assumption above.

We denote the instantaneous utility of an individual as U_j^i, where $j \in \{Y, O\}$ denotes whether he is young (Y) or old (O), and where $i \in \{D, N\}$ denotes whether the old member of his family (namely himself if $j = O$ or his parent if $j = Y$) is dependent (D) or not (N). At the time they take decisions, young agents do know the dependency status of their parent.

The utility a young agent obtains from consuming c is represented by the increasing and strictly concave utility function $u(c)$. If an agent remains autonomous in old age, his preferences for consumption do not change and are represented by the same utility function $u(d)$, where d measures consumption when old. The utility function of a dependent elderly is denoted by $v(d, e)$.[7] We assume that v is increasing and concave in both consumption d and informal help e. We discuss the function $v(.)$ later in this section, once all notations have been introduced.

Starting with the parents, we have that

$$\begin{aligned} U_O^N &= u(d), \\ U_O^D &= v(d, e). \end{aligned}$$

Individuals do not work in the second period of their life, but all receive the same income $\bar{p}$. This income can be considered as a second-period endowment, as in Pestieau and Sato (2006, 2008), or as a pension financed by (unmodelled) contributions, for instance. The important assumption is that $\bar{p}$ is the same for all.[8] We then have that $d = \bar{p}$ for autonomous parents, while dependent parents may also receive financial transfers from their child and from the government, so that their consumption level is given by

$$d = \bar{p} + f + b.$$

Old agents take no decision but simply consume their income and benefit from informal help, if any.

As for the children, we have that

$$\begin{aligned} U_Y^N &= u(c), \\ U_Y^D &= u(c) + \alpha v(d, e), \end{aligned}$$

where $\alpha > 0$ is an altruism parameter, identical across children.

[7] Finkelstein et al. (2013) show that the marginal utility from consumption varies with health status.

[8] We leave for future research the case where old agents differ in income, and where children and parental income within a family are correlated.

We now turn to the determination of the children consumption level, and of the social LTC transfers. Young agents differ in their productivity w, which is distributed over $[w_-, w_+]$ according to the cdf $F(w)$, with the median productivity, w_{med}, lower than the average productivity, $\bar{w}$. They all supply a unit of labor time in return for a gross wage equal to their productivity w. Wage income is taxed at a proportional rate $\tau \in [0, 1]$, with the proceeds used to finance the lump sum transfer b to all dependent agents (a pay-as-you-go social LTC transfer program). The government budget balance equation is given by[9]

$$b(\tau) = \frac{\tau \bar{w}}{\pi}.$$

Young agents with dependent parents choose how to allocate the remainder of their time, $T - 1$. They choose the amount of time e that they spend helping their parent (informal help, with $0 \leq e \leq T - 1$). The time spent neither working in the taxed sector nor helping one's parent is spent earning an income equal to one's productivity and not taxed (for instance by moonlighting in the underground economy, or because overtime work is not subject to social insurance contributions, as was the case in France between 2007 and 2012). The after-tax income of a young agent of productivity w providing an amount e of informal help to his dependent parent is then equal to[10]

$$w(1 - \tau) + (T - 1 - e)w = w(T - e - \tau).$$

The important characteristic of this formulation is that informal help has an opportunity cost for young agents, so that informal help is more costly for more productive agents.

Independently of their choice of e, young agents can also help their dependent parent financially by making a monetary transfer f. Young agents then consume

$$c = w(T - \tau - e) - f.$$

Before turning to the choice of private (formal and informal) and social transfers to dependent individuals, we introduce several assumptions used throughout the paper. We take as numeraire the market wage of a professional helper of dependent people (such as a nurse or a specialized caretaker). Hence, e denotes both the amount of time spent with dependent parents and the market cost of this time, if it were spent by a professional care taker.

[9] Assuming away labor income tax distortions biases the model in favor of the introduction of social LTC transfers - see footnote 16 for the impact of introducing distortions.

[10] This formulation will prove easier to handle than the alternative where all labor income is taxed, so that after-tax income is $w(T - e)(1 - \tau)$. This does not change our results qualitatively.

Pinquart and Sorensen (2002) show that dependent individuals in general prefer informal support, or a combination of formal and informal support, to formal support, at least for short term needs. We model this observation as the following assumption.

Assumption 1 *The instantaneous utility of dependent individuals has the form*

$$v(d,e) = \phi\left(d + \beta e\right),$$

with $\beta > 1$ and the function $\phi(.)$ increasing and strictly concave.

Assumption 1 has two main consequences. First, the premium that dependent parents put on informal help from their child (compared to formal help) is a constant β:

$$v_e\left(d,e\right) = \beta v_d\left(d,e\right).$$

We call the parameter β the intensity of the preference for informal help of dependent parents. In our setting, a dependent elderly values one more hour of informal family help as much as β hours of a professional care taker (or β times the market cost of one hour of professional help). The parameter β is identical across agents.

Second, Assumption 1 implies that monetary and informal (time) help are *substitutes*, that is $v_{ed}\left(d,e\right) = \beta\phi''\left(.\right) < 0$. This substitutability is borne out by many recent papers like Bonsang (2007, 2009), Charles and Sevak (2005), Van Houtven and Norton (2004).

Finally, we will make use of the following assumption:[11]

Assumption 2 *The coefficient of relative risk aversion is lower than one for (a) autonomous agents*

$$-\frac{u''(c)c}{u'(c)} \leq 1,$$

as well as (b) for dependent agents

$$-\frac{\phi''(x)x}{\phi'(x)} \leq 1.$$

We now turn to the determination of private (formal and informal) and social LTC transfers.

[11]Karagyozova and Siegelman (2012) review the empirical literature on relative risk aversion. They report several studies which find a coefficient of relative risk aversion lower than one. Hansen and Singleton (1983) evaluate it to be between $[0.35, 1]$; Holt and Laury (2002) elicit the distribution of coefficients of relative risk aversion and find that 64% of respondents had a coefficient comprised between 0.15 and 0.97. Chetty (2006) finds that "using 33 sets of estimates of wage and income elasticities, the mean implied value of (the coefficient of relative risk aversion) is 0.71, with a range of 0.15 to 1.78 in the additive utility case." Assumption 2 is then reasonable.

3 The determination of private (formal and informal) and social LTC transfers

All agents (young and old) vote over the size $\tau \in [0,1]$ of the social LTC transfer program. Young individuals observe the result of the vote and then decide privately the amounts of formal and informal transfers to their dependent parent. The dependency status of parents is known to all both when voting and when taking private decisions. We assume that voting takes place in each period, so that second period utility of agents young at the time of voting is not affected, but will be determined by the next vote, which takes place when they reach old age.[12]

It is straightforward to see that young agents with an autonomous parent favor no LTC transfers at all ($e = f = \tau = 0$). Similarly, old agents who are autonomous are indifferent about the level of τ, since they do not pay labor income taxes nor receive any transfer. We thus focus in the next two subsections on young individuals with a dependent parent and on dependent elderly voters.

3.1 Children with a dependent parent

Using the government budget constraint together with the definition of consumption levels c and d, we obtain that

$$U_Y^D = u(w(T - \tau - e) - f) + \alpha v\left(\bar{p} + f + \tau\bar{w}/\pi, e\right).$$

We solve the model backward, starting with the optimal formal and informal transfers as a function of the exogenously chosen social transfer program size τ. The FOCs for formal and informal help are given by

$$FOC_f = \frac{\partial U_Y^D}{\partial f} = -u'(c) + \alpha v_d\left(\bar{p} + f + \tau\bar{w}/\pi, e\right) \leq 0, \tag{1}$$

$$\begin{aligned} FOC_e &= \frac{\partial U_Y^D}{\partial e} = -wu'(c) + \alpha v_e\left(\bar{p} + f + \tau\bar{w}/\pi, e\right) \\ &= -wu'(c) + \alpha\beta v_d\left(\bar{p} + f + \tau\bar{w}/\pi, e\right) \leq 0 \end{aligned} \tag{2}$$

Comparison of the two FOCs allows us to state the following proposition, where the most-preferred levels of variables are denoted as $f^*(\tau, w)$ and $e^*(\tau, w)$, which we abbreviate as f^* and

[12] The assumption that the result of a vote would hold for decades does not seem reasonable to us. The literature on the political economy of pensions often assumes that voting takes place once and for all (see for instance, Casamatta et al., 2000 and Cremer et al., 2007), but this assumption is a *pis-aller* to explain the emergence of pay-as-you-go social transfer schemes in the absence of altruism. The presence of altruism in our model makes the unpalatable assumption of voting once and for all not necessary.

e^* when this shortened notation is non ambiguous.

Proposition 1 *Young agents with a dependent parent*

(a) prefer to give financial rather than informal help ($f^* > 0$, $e^* = 0$) if their wage is larger than the intensity of the preference for informal help of dependent parents ($w > \beta$),

(b) prefer informal to formal help ($f^* = 0$, $e^* > 0$) otherwise ($w < \beta$).

(c) Except when $w = \beta$ (where they are indifferent as to the specific mix of formal and informal help but rather care about the monetary value of total help using their opportunity cost for informal help, $f + we$), they never give simultaneously formal and informal help, i.e. ($f^* > 0$, $e^* > 0$) is not an equilibrium.

Proof. (a) For any pair $(e > 0, f > 0)$, we have that $FOC_f > FOC_e$ when $w > \beta$. Choosing $f^* > 0$, we have that $FOC_f = 0 > FOC_e$ for any e, so that $e^* = 0$ and ($f^* > 0$, $e^* = 0$) is an equilibrium. There is no equilibrium with $e^* > 0$ since $FOC_f > FOC_e = 0$ implies that there is no equilibrium value for f.

(b) is proven likewise for $\beta > w$.

(c) The proofs of (a) and (b) show that ($e^* > 0$, $f^* > 0$) cannot be an equilibrium when $w \neq \beta$. If $\beta = w$, then $FOC_f = FOC_e$ for all pair (e, f). Equalizing FOC_f to zero and using Assumption 1, we obtain that

$$u'(w(T-\tau) - [f + we]) = \alpha\phi'(\bar{p} + \tau\bar{w}/\pi + [f + we]),$$

so that $f + we$ has to be set at the adequate level. ■

Children of a dependent elderly compare their opportunity cost of giving informal help, w, with the intensity of the preference of the elderly for informal help, β, and choose informal help if they are sufficiently unproductive ($w < \beta$) and formal help otherwise ($w > \beta$). The only case where they are indifferent between both is when the opportunity cost of informal help equals its benefit for the parent ($w = \beta$).

We now perform some comparative statics analysis of the most-preferred level of family help, starting with the informal help provided by low productivity agents.

Proposition 2 *For all children of dependent parents with $w < \beta$, the most-preferred level of informal help, e^*,*

(a) decreases with w ,

(b) increases with β

(c) decreases with τ. Moreover, informal help is a perfect substitute to social LTC transfer ($de^*/d\tau = -1$) when $\beta = \bar{w}/\pi$, and decreases faster than τ ($de^*/d\tau < -1$) when $\beta < \bar{w}/\pi$ and more slowly than τ ($de^*/d\tau > -1$) when $\beta > \bar{w}/\pi$.

Proof. (a) By the implicit function theorem applied to (2) with $f = 0$, we obtain

$$sign\left(\frac{de^*}{dw}\right) = sign\left(-u'(c) - cu''(c)\right)$$

which is negative or null under Assumption 2(a).

(b) Proceeding in the same manner with β, using Assumption 1, we obtain that

$$sign\left(\frac{de^*}{d\beta}\right) = sign\left(\alpha\left[\phi'\left(\bar{p} + \tau\bar{w}/\pi + \beta e^*\right) + \beta e^*\phi''\left(\bar{p} + \tau\bar{w}/\pi + \beta e^*\right)\right]\right),$$

which is strictly positive given Assumption 2(b).

(c) Proceeding in the same manner with τ, using Assumption 1, we obtain that

$$\frac{de}{d\tau} = -\frac{w^2u''(c) + \alpha\left(\bar{w}/\pi\right)\beta\phi''(.)}{w^2u''(c) + \alpha\beta^2\phi''(.)} < 0. \tag{3}$$

The rest of the proof is straightforward. ■

The intuition for part (a) is that an increase in w has two countervailing effects on informal family help: it increases the monetary opportunity cost of informal help (inducing less help) but it also decreases the marginal utility of consumption when young (inducing more help). Assumption 2 makes sure that the latter effect is low enough that the net impact of a larger income on informal help is negative.

The intuition for part (b) is similar in that a larger value of β also has two countervailing impacts on e^*: increasing β has both a positive direct impact on the marginal utility of informal help for the dependent parent but also, given Assumption 1, a negative indirect impact (since a larger value of β for a given informal help amount e translates into a lower marginal utility of the dependent agent). Assumption 2(b) guarantees that the former impact is larger than the latter, so that a larger value of β translates into more such help given by agents with $w < \beta$.

Part (c) first establishes that the social LTC transfer crowds out the supply of informal help: as τ increases, the marginal utility cost of giving help increases, while the marginal benefit decreases. When the financial return of the social transfer tax, $\bar{w}/\pi$, is equal to the intensity of the preference of the dependent elderly for informal help, β, the two forms of help are perfect substitutes. When $\beta < \bar{w}/\pi$, (resp., $\beta > \bar{w}/\pi$) the return of social transfer is larger (resp., smaller) than the "psychological return" of informal help, and the amount of informal help e^* provided decreases more (resp. less) than the value of the social transfer contribution rate, τ.

We have a similar (but not equivalent) proposition for the comparative statics analysis of formal help given by more productive agents.

Proposition 3 *For all children of dependent parents with $w > \beta$, the most-preferred level of formal help, f^*,*
(a) increases with w,
(b) is independent of β,
(c) decreases with τ. Moreover, formal help is a perfect substitute to social LTC transfer ($df^*/d\tau = -\bar{w}/\pi$, so that the total monetary value of help $\tau\bar{w}/\pi + f^*$ is unchanged) when $w = \bar{w}/\pi$, and decreases faster than τ ($df^*/d\tau < -\bar{w}/\pi$, so that the total monetary value of help $\tau\bar{w}/\pi + f^*$ decreases) when $w > \bar{w}/\pi$ and more slowly than τ ($df^*/d\tau > -\bar{w}/\pi$, so that the total monetary value of help $\tau\bar{w}/\pi + f^*$ increases) when $w < \bar{w}/\pi$.

Proof. (a) By the implicit function theorem applied to (1) with $e = 0$, we obtain

$$sign\left(\frac{df^*}{dw}\right) = sign\left(-(T-\tau)u''(c)\right) > 0.$$

(b) Immediate since (1) does not depend on β when $e = 0$.
(c) Proceeding similarly with respect to τ, we obtain

$$\frac{df^*}{d\tau} = -\frac{wu''(c) + \alpha\left(\bar{w}/\pi\right)\phi''(.)}{u''(c) + \alpha\phi''(.)} < 0. \tag{4}$$

The rest of the proof is straightforward ■

The intuition for part (a) is that a larger child's income decreases his marginal cost of giving, inducing him to transfer more money to his dependent parent. Propositions 2(a) and 3(a) are supported by several empirical papers, which are listed in section 1. The intensity of the preference for informal help of the parent has no impact on the amount of financial transfer made by the child as long as he is sufficiently productive that he prefers making a formal rather than an informal transfer to his parent.

Formal help is a substitute to social LTC transfers: as τ increases, the marginal cost of giving increases while the marginal benefit decreases, inducing a lower level of direct financial help. When $w = \bar{w}/\pi$, both forms of help are perfect substitutes (because they have the same marginal cost and benefit for the helper), so that the total monetary value of the transfers, $\tau\bar{w}/\pi + f^*$, remains unchanged when τ increases. When $\bar{w}/\pi < w$, (resp., $\bar{w}/\pi > w$) social transfers have a lower (resp., higher) return than direct transfers, so that total monetary transfers decrease (resp., increase) with τ.

An important lesson to draw from these two propositions is that both the monetary ($f^* + e^*$) and the psychological ($f^* + \beta e^*$) value of family help are at their minimum for middle income agents–i.e., agents with $w = \beta > 1$ (recall the numeraire is the wage of a professional care taker). Indeed, every agent with $w < \beta$ chooses $e^* > 0$ which is decreasing in w up to $w = \beta$ while every agent with $w > \beta$ prefers $f^* > 0$ which is increasing in w.
Note also that agents with $w = \beta$ wish to help (formally or informally) their elderly parent provided that the following condition is satisfied

$$u'(\beta(T - \tau)) < \alpha\phi'(\bar{p} + \tau\frac{\bar{w}}{\pi}). \tag{5}$$

This condition requires that (i) agents are sufficiently altruistic (α large enough), (ii) that dependent elderly resources in the absence of family help are low enough (low $\bar{p}$, τ and low return of social transfer $\bar{w}/\pi$) and (iii) that help is not too costly for the child (thanks to a low contribution rate τ, a large amount of available time T and a large productivity $w = \beta$).

When condition (5) is not satisfied, the middle class prefers not to provide any (formal or informal) family help, as exemplified in Figure 1.[13]

We now move to the first stage of the model and study the individually most-preferred social transfer contribution rate τ of young agents with a dependent parent. The FOC for τ is given by

$$FOC_\tau = \frac{\partial U_Y^D}{\partial \tau} = -wu'(c) + \alpha\frac{\bar{w}}{\pi}v_d(\bar{p} + f + \tau\bar{w}/\pi, e) = 0, \tag{6}$$

while the FOCs for f and e remain given by (1) and (2).

We obtain the following proposition.[14]

[13]Figures are based on the following assumptions: $u(x) = 2\sqrt{x}$, $v(d, e) = ln[d + \beta e]$, $\pi = 0.5$, $\bar{w} = 5$, $T = 2$, $\bar{p} = 4$, $\beta = 5$, $\tau = 0$ and $\alpha = 1$.

[14]In order to focus on empirically relevant situations and not to multiply cases, we assume from now on that no young agent with a dependent parent most prefers $\tau^* \geq 1$, which could in theory occur in our setting with

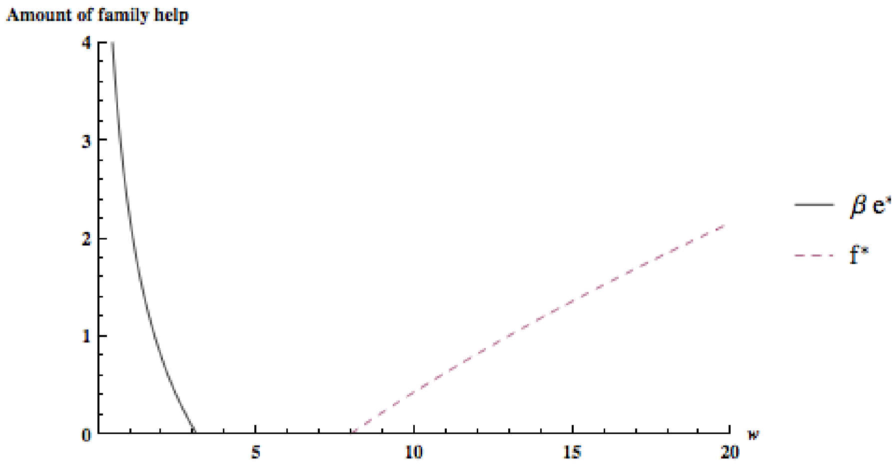

Figure 1: Formal and informal help as a function of income when condition (5) is not satisfied.

Proposition 4 *Among the children with a dependent parent,*

(a) those with high productivity prefer to make only direct formal transfers: $f^* \geq 0$, $\tau^* = e^* = 0$ *if* $w > \max[\beta, \bar{w}/\pi]$.

(b) those with low productivity prefer exclusively social transfers if $\bar{w}/\pi > \beta$*:* $\tau^* \geq 0$, $f^* = e^* = 0$ *if* $w < \max[\beta, \bar{w}/\pi] = \bar{w}/\pi$.

(c) those with low productivity prefer exclusively informal help if $\bar{w}/\pi < \beta$*:* $e^* \geq 0$, $f^* = \tau^* = 0$ *if* $w < \max[\beta, \bar{w}/\pi] = \beta$.

(d) A necessary and sufficient condition for every child to have a strictly positive most-preferred transfer is

$$u'(T \max[\beta, \bar{w}/\pi]) < \alpha\phi'(\bar{p}).$$

Proof. See Appendix ■

The intuition for Proposition 4 is clear. Very productive children prefer to make a direct financial transfer to their elderly parent, because the other two forms of transfers have a higher marginal cost (opportunity cost for informal help, tax bill for social transfer). Less productive children shun direct financial transfers and compare the returns of the other two forms of help (which have the same marginal cost), namely the intrinsic preference for informal help of

untaxed labor income $w(T-1)$. This condition is satisfied if

$$wu'(w(T-1)) > \alpha\frac{\bar{w}}{\pi}\phi'(\bar{p} + \frac{\bar{w}}{\pi}).$$

dependent parents, β, and the financial return of social transfers, $\bar{w}/\pi$. They most-prefer the transfer with the highest return. Observe that these returns are independent of the individual characteristic, namely the productivity level w.

Overall, if $\beta > \bar{w}/\pi$, social transfers are dominated by informal help for all children with dependent parents, and the choice between f and e, as well as the comparative statics analysis of their most-preferred form of help, remains as described in Propositions 2 and 3: agents with income $w < \beta$ provide informal help while those with $w > \beta$ prefer formal help.

In that case, there cannot be a majoritarian support for social transfers, since no young agent supports it. A large intrinsic preference for informal help may then explain why we observe so little social LTC transfers. To go beyond this result, we assume from now on that $\beta < \bar{w}/\pi$. In that case, informal help is dominated by the social transfer, so that no one most-prefers a positive value of e, and the most-preferred form of help is either social transfers (for children with $w < \bar{w}/\pi$) or direct financial transfers (for children with $w > \bar{w}/\pi$). The next proposition studies the comparative statics properties of τ^*.

Proposition 5 *Assume that $\beta < \bar{w}/\pi$. Individuals with $w < \bar{w}/\pi$ are such that:*
(i) Their most-preferred social LTC contribution rate τ^* (a) decreases with w, (b) is independent of β, (c) decreases with π.
(ii) Their most-preferred social LTC transfer, b^*, is larger than $\beta e^*(0, w)$, where $e^*(0, w)$ is their optimal informal help amount when taxation is not available.

Proof. See Appendix ■

The intuition for part (i) (a) is similar to that of Proposition 2(a), as an increase in w has two countervailing effects on the preferences for τ: it increases the monetary opportunity cost of the social transfer (inducing a lower value of τ) but also decreases the marginal utility of consumption when young (inducing a higher value of τ). Assumption 2(a) makes sure that the latter effect is small enough that the net impact of a larger income on the most-preferred amount of social transfer is negative. As for the intensity of the preference for informal help by the elderly, it plays no role in the determination of τ^*, as long as $\beta < \bar{w}/\pi$ so that informal help is dominated by social transfers. Finally, as the fraction of dependent individuals increases, the preferred tax rate decreases. On the one hand, the return agents can expect to obtain from taxation decreases, increasing the cost of a given social transfer. On the other hand, raising

π increases the marginal utility from obtaining this transfer when old, for a given τ. Under assumption 2(b) the former effect dominates the latter, so that τ decreases with π.

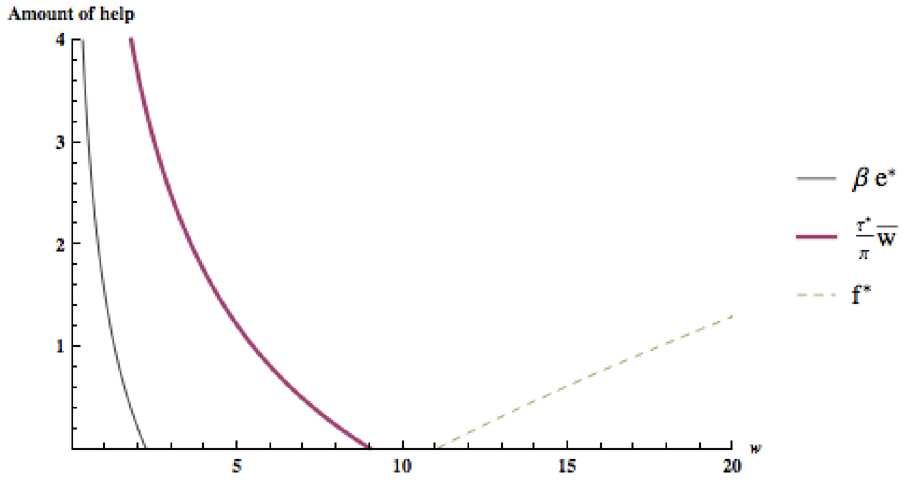

Figure 2: Formal, informal help and social LTC care transfers as a function of income, for $\alpha = 0.85$.

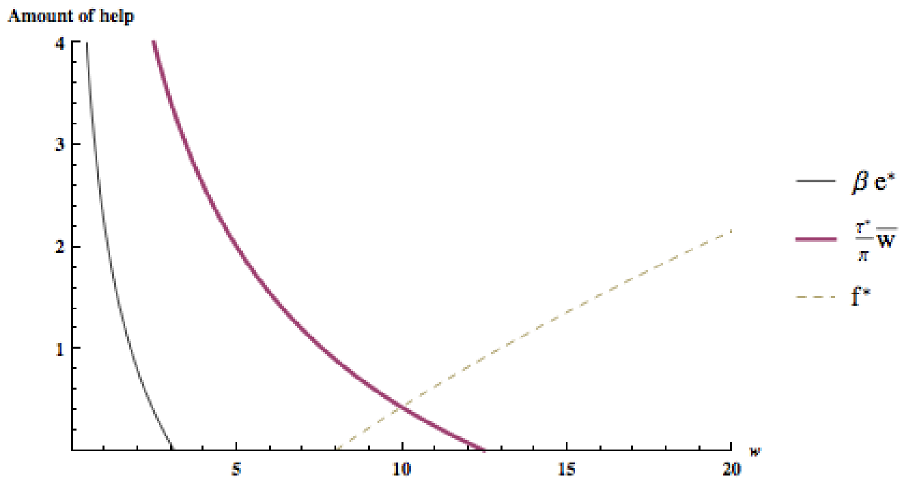

Figure 3: Formal, informal help and social LTC care transfers as a function of income, for $\alpha = 1$.

The intuition for part (ii) is that the social program offers a better return than informal help when $\beta < \bar{w}/\pi$, so that agents are willing to contribute more to the social program than they would give informal help if the social program were not available. The dependent parents of those agents then benefit from the introduction of the social program (at the most-preferred allocation of their child) because they receive more help, both at the intensive margin (as shown in part (ii)) but also at the extensive margin, as the set of middle-class agents who do not help their dependent parent shrinks – see Figure 2– (and may even disappear, as in Figure 3) when the social program is made available.

We now turn to the preferences of dependent parents for the social transfer program.

3.2 Dependent parents

Using the government budget constraint as well as the formula for second-period consumption, the utility function of an elderly dependent becomes

$$U_O^D = v\left(\bar{p} + f^*(\tau, w) + \tau\frac{\bar{w}}{\pi}, e^*(\tau, w)\right),$$

where the amount of family help received (e^* and f^*) is chosen by his child of productivity w. The elderly dependent determines his most-preferred social transfer contribution rate τ^* by taking into account the crowding out of family help by the social transfer (see Propositions 2(c) and 3(c)). We obtain the following proposition.[15]

Proposition 6 *Assume $\beta < \bar{w}/\pi$.*
(a) Dependent parents of a low productivity child ($w < \beta$) most-prefer $\tau^* = 1$.
(b) Dependent parents of a child with income $w \in [\beta, \bar{w}/\pi]$ most-prefer $\tau^* = 1$.
(c) There exists a threshold productivity level $\breve{w} > \bar{w}/\pi$ such that dependent parents of a child with income $w \in [\bar{w}/\pi, \breve{w}[$ most-prefer $\tau^* = 1$ while those with a child with income $w \geq \breve{w}$ most-prefer $\tau^* = 0$.

Proof. See Appendix. ■

Part (a) of Proposition 6 tackles the case where the child's productivity is low enough ($w < \beta$) that family help is provided in an informal way. Social transfers crowd out informal help, and it may then be surprising that parents prefer $\tau^* = 1$, for two reasons. First, it is precisely the lowest income children who give the most informal help (see Proposition 2 (a)). Second, Proposition 2 (c) has established that $de^*/d\tau < -1$ when $\beta < \bar{w}/\pi$–i.e., that the crowding out is intense since informal help decreases faster than τ. Despite this, Proposition 6 (a) shows that the total value of the transfers received, $\tau\bar{w}/\pi + \beta e^*$, increases with τ as soon as $\beta < \bar{w}/\pi$.

[15] The case where $\beta > \bar{w}/\pi$ is available upon request but not reported here since the majority chosen value of τ is zero in that case.

Dependent parents then prefer to totally crowd out informal help by their child and most-prefer the highest admissible value of τ.[16]

Parts (b) and (c) study the case where any help from the family takes the form of monetary transfers (because $w > \beta$). The intuition for part (b) is similar to the one developed in the previous paragraph: even though social transfers crowd out family help, the total value of the transfer received, $\tau\bar{w}/\pi + f^*$, increases with τ when $w < \bar{w}/\pi$ (see Proposition 3(c)) and old dependents prefer to totally crowd out family help and most-prefer $\tau^* = 1$. When $w > \bar{w}/\pi$ as in part (c), the crowding out of family by social transfers happens so fast that the total transfer received decreases. When τ is large enough, family help is totally crowded out, and the (social) transfer increases with τ, so that dependent parents have V shaped preferences over τ. When w is lower than a threshold $\breve{w}$ (defined in the Appendix), the maximum monetary transfer received from the family (when $\tau = 0$) is small enough that old dependents prefer to totally crowd out family help and choose $\tau^* = 1$. When w is larger than $\breve{w}$, dependent parents prefer the maximum family transfer (corresponding to $\tau = 0$) to the maximum social transfer (when $\tau = 1$). In a nutshell, dependent parents prefer the largest social LTC transfer system ($\tau^* = 1$), even at the cost of totally crowding out (in)formal family help, except if their child is rich enough ($w > \breve{w}$) that he transfers more money than the maximum social transfer available.

Note that $\breve{w} > \bar{w}/\pi$ –i.e., the threshold child's income above which agents in a family with a dependent member prefer $\tau = 0$ is larger for parents than for children, because the former do not consider the utility cost for their child of paying the contribution but focus on the benefit they receive. In other words, families where the child income is in-between $\bar{w}/\pi$ and $\breve{w}$ disagree on the form that the LTC transfer should take, with children preferring formal help from the family while parents prefer social transfers.

We now study the majority voting equilibrium over τ.

3.3 The majority voting equilibrium

As stated at the beginning of Section 3, autonomous parents are indifferent as to the value of τ, so that we suppose that they abstain and do not vote.[17] Voters are then composed of three

[16] Introducing labor income tax distortions would have two effects: (i) shrinking the set of agents who most-favor positive contribution rates and (ii) for those who do, the most-preferred value of τ would be the interior one maximizing the tax proceeds.

[17] Alternatively, we could assume that they throw a dice to determine which value of τ to vote for. This would not affect our results.

groups: young with dependent parents, young with autonomous parents and old dependent agents. The next lemma looks at the relative size of these groups.

Lemma 1 *No group forms a majority by itself when voting provided that* $\pi > 1/3$.

Proof. With a unitary mass of agents in each generation and two generations voting simultaneously, the total mass of voters is $1+\pi$ (all young plus the dependent elderly). Young voters with a dependent parent, and dependent parents, never form a majority by themselves since $\pi < (1+\pi)/2 \Leftrightarrow \pi < 1$. The condition for children of autonomous parents not to form a majority is $1-\pi < (1+\pi)/2 \Leftrightarrow \pi > 1/3$. ■

Since children with autonomous parents prefer no social insurance transfers, a necessary condition for a positive value of τ to emerge from majority voting is that $\pi > 1/3$. This condition is empirically reasonable. For instance, according to Brown and Finkelstein (2009), in the US, "the probability that a 65 year old individual will use a nursing home at some point in his or her life is quite substantial, with estimates ranging from 35 to nearly 50 percent".[18] We assume from now on that $\pi > 1/3$.

The main difficulty with proving the existence of a majority-voting equilibrium is due to the fact that the preferences of the old dependent with $w \geq \bar{w}/\pi$ are not single peaked (see proposition 6(c)). We then define a global (resp., local) Condorcet winner, τ^{GCW} (resp., τ^{LCW}) as a value of τ that is preferred to any other value $\tau \in [0,1]$ (resp., to any value of $\tau \in [0,1]$ in a neighborhood of τ^{LCW}) by a majority of voters.

The next proposition studies the conditions under which the absence of social LTC transfers ($\tau = 0$) is a (global or local) Condorcet winner. Recall that we concentrate on the case where $\beta < \bar{w}/\pi$, as this constitutes a necessary condition for young agents to prefer the LTC social transfer to (in)formal help.

Proposition 7 *When* $\beta < \bar{w}/\pi$ *and* $\pi > 1/3$, $\tau = 0$ *is*
(a) the global Condorcet winner if

$$1 + \pi \left[1 - \left(F(\bar{w}/\pi) + F(\breve{w})\right)\right] \geq \frac{1+\pi}{2}, \tag{7}$$

(b) a local Condorcet winner if

$$1 - \pi + 2\pi \left[1 - F(\bar{w}/\pi)\right] \geq \frac{1+\pi}{2}. \tag{8}$$

[18] See also Torjman (2013).

Proposition 7 studies the conditions under which $\tau = 0$ maximizes (locally or globally) the utility of a majority of voters. The RHS of (7) measures the total mass of voters while its LHS gives the mass of agents who prefer $\tau = 0$ to any other value of $\tau \in [0, 1]$,

$$(1 - \pi) + \pi(1 - F(\bar{w}/\pi)) + \pi(1 - F(\breve{w})),$$

where the first term measures the mass of children with autonomous parents, the second the mass of young agents with dependent parents who prefer giving formal family help to paying for social transfers (see Proposition 4(a)), and the third term the mass of dependent parents who prefer no social transfer in order not to crowd out the formal family help they receive (see Proposition 6(c)).

To obtain the (weaker) condition (8) for $\tau = 0$ to be a local Condorcet winner, we add to the LHS of (7) the mass of individuals whose utility is locally decreasing in τ from $\tau = 0$, even though $\tau = 0$ is not their most-preferred tax rate – namely, the parents whose child's income $w \in [\bar{w}/\pi, \breve{w}]$ (see equation (12) in the Appendix).

We now show that conditions (7) and (8) can actually be satisfied. The LHS of (7) is minimum when $F(\bar{w}/\pi)$ and $F(\breve{w})$ are both close to one, so that it is close to $1 - \pi$. Observe that $1 - \pi > (1 + \pi)/2$ implies that $\pi < 1/3$. So, as long as $F(\bar{w}/\pi) < F(\breve{w}) < 1$, there exist values of π slightly larger than 1/3 (i.e., satisfying Lemma 1) such that $\tau = 0$ is a global Condorcet winner. Condition (8) is easier to satisfy since the LHS of (8) is larger than the LHS of (7) (given that $F(\bar{w}/\pi) < F(\breve{w})$) so that, by the same reasoning as before, this condition is satisfied for values of π slightly larger than 1/3. It is indeed quite intuitive that a low prevalence of dependency (a low π) is conducive to an equilibrium with no social transfers. Notwithstanding this remark, increasing π has an ambiguous impact on the relative share, among voters, of those who favor $\tau = 0$. Increasing π reduces the return of the social LTC program, increasing the fraction of voters in dependent families who most-prefer $\tau = 0$.[19] At the same time, there are fewer children of autonomous parents who oppose any taxation. Although the net impact of a higher π on the LHS of (7) is positive, this is also true for the RHS–i.e., the fraction of agents who cast a vote (since fewer autonomous elderly abstain). It is then by no way clear how condition (7) is affected by an increase in π. A similar reasoning applies to condition (8). We obtain numerically, using a lognormal distribution of income, that both conditions are satisfied

[19]It is easy to show, using the implicit function theorem on (1), that $\breve{w}$ decreases with π.

provided that π is not too large (the threshold level of π being of course higher for condition (8) than for (7)).

Proposition 7 shows that the absence of social LTC transfers can be a Condorcet winner even when $\beta < \bar{w}/\pi$ –i.e., in situations where some (young and old) agents most-prefer social transfers rather than informal family help. The majority coalition is then made of children of autonomous parents together with rich families with a dependent elderly.

The absence of social LTC transfer can also be a local, but not a global, Condorcet winner, meaning that $\tau^* = 0$ can be defeated by a value of τ large enough that, for dependent parents of middle-range income children ($w \in [\bar{w}/\pi, \breve{w}]$), (i) formal help is totally crowded out, and (ii) the social LTC transfer is larger than the formal family help that would have been received in the absence of social LTC transfers.

The next proposition characterizes the (global or local) Condorcet winning value of τ when $\beta < \bar{w}/\pi$, and condition (7) is not satisfied.

Proposition 8 *When $\beta < \bar{w}/\pi$, $\pi > 1/3$ and condition (7) is not satisfied, we obtain a (interior) Condorcet winner in two cases.*

(a) (i) A sufficient condition for the existence of a global Condorcet winner is that $df^*/d\tau < 0$ is monotonically decreasing in w when $f^* > 0$. (ii) This condition is satisfied with logarithmic utilities. (iii) Under (i), $0 < \tau^{GCW} < 1$. The decisive voter is a young, with a dependent parent, whose income $w^{GCW} < \bar{w}/\pi$ is such that

$$F(\breve{w}) + F(w^{GCW}) = \frac{1+\pi}{2\pi}.$$

(b) A local Condorcet winner $0 < \tau^{LCW} < 1$ exists if τ^{LCW} satisfies the following conditions: (i) $f^*(w_+, \tau^{LCW}) = 0$ and (ii) the child of a dependent parent who most-prefers τ^{LCW} and whose income level, denoted by w^{LCW}, is such that

$$1 + F(w^{LCW}) = \frac{1+\pi}{2\pi}. \tag{9}$$

(c) In both cases, the equilibrium allocation among the children with a dependent parent is such that (i) those with a sufficiently low productivity level complement social transfers with informal help, (ii) those with a productivity level high enough complement social transfers with formal help, and (iii) those with intermediate productivity levels do not offer any family help and rely exclusively on the public transfer.

Proof. See Appendix. ■

The proof of part (a) consists in forming four exogenous (i.e., independent of the value of τ) groups of voters in families with a dependent parent, according to their age (child or parent) and to whether the child in the family prefers informal to formal help ($w < \beta$) or the opposite ($w > \beta$). We then show that preferences satisfy the single-crossing condition (see Gans and Smart, 1996) for these four groups. This is always the case for children of dependent parents, and for dependent parents of low income ($w < \beta$) children. The necessary and sufficient condition for preferences of dependent individuals with rich children ($w > \beta$) to satisfy the single-crossing condition is that $d^2 f^*/d\tau dw < 0$, so that the formal help f^* received by dependent parents decreases faster with τ for parents of richer children. We show that this reasonable condition is indeed satisfied with logarithmic utilities. When this condition is satisfied, the decisive voter is a child of a dependent parent with an income level w^{GCW} such that one half of the polity is formed of poorer children of dependent parents (a mass $\pi F(w^{GCW})$) together with dependent parents of children with income below $\breve{w}$ (a mass $\pi F(\breve{w})$), who both prefer a larger value of τ. In other words, this equilibrium is sustained by the opposition between sufficiently poor members of families with a dependent parent, and the rest of the voting population (who wants a lower value of τ).

Part (b) shows that a strictly positive value of τ may be a local Condorcet winner provided that it totally crowds out formal help by the family. In that case, the utility of all dependent parents is locally increasing in τ. We then add to the measure of dependent parents (π) the measure of children of dependent parents whose income is low enough that they would favor a larger value of τ ($\pi F(w^{LCW})$). If the sum of both groups represents one half of the voters (see condition (9)), then $\tau^{LCW} > 0$ is indeed a local Condorcet winner. Note that the productivity of the decisive voter, w^{LCW}, decreases with π but we cannot conclude that the equilibrium tax level τ^{LCW} will be larger, because a larger value of π also decreases the return of the social LTC program. Observe that, in both cases (global and local equilibrium), the Condorcet winning value of τ is interior, even though older agents have corner solutions ($\tau = 0$ or $\tau = 1$).

In both cases, the equilibrium allocation is such that children of dependent parents whose income is sufficiently different from the decisive voter's income complement the social transfers with informal (for low income levels) or formal (for high income levels), while intermediate income levels (close to the decisive voter's income) rely exclusively on social transfers.

4 Conclusion

Our model generates a pattern of family help which conforms to what is empirically observed: low income children provide informal help to their dependent parent while richer children provide formal help. Informal help is decreasing in the child's income, while formal help is increasing. We also obtain that children with income in a middle range may not provide help at all.

We then highlight several reasons why social LTC transfers may not be offered at the majority voting equilibrium. First, if the intrinsic preference of dependent parents for informal help is large enough, then social transfers are dominated by informal help. Second, if the probability of becoming dependent when old is lower than one third, the children of autonomous parents are numerous enough to oppose democratically the introduction of social LTC transfers. Third, even when none of the first two conditions is satisfied, the majority voting equilibrium may entail no social transfers, especially if the probability of becoming dependent when old is not far above one third. We identify conditions for the existence of a local Condorcet winner with no social transfers. This local majority equilibrium may be empirically relevant whenever new programs have to be introduced at a low scale before being eventually ramped up.

Our model is a very stylized representation of reality, which could be improved in several directions. Parents as well as children could differ in income. The social program could consist in an insurance program rather than a transfer program. In that case, agents would vote over this program before knowing whether their parent will become dependent. This should strengthen the demand for the social programs, unless a private insurance program is also available. Introducing distortions from taxation should lower the demand for public intervention. Agents could also differ in risk (probability of becoming dependent), for instance because of exogenous differences in life expectancies, and in the existence of altruistic children. Note that several of these factors have been introduced in De Donder and Pestieau (2015), but their setting does not differentiate formal from informal family help.

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Appendix

Proof of Proposition 4

(a) $w > \bar{w}/\pi$ implies that $FOC_f > FOC_\tau$ $\forall(\tau, f, e)$ while $w > \beta$ implies that $FOC_f > FOC_e$ $\forall(\tau, f, e)$. If $FOC_f \leq 0$ for $\tau = f = e = 0$ then $FOC_\tau < 0$ and $FOC_e < 0$ and $\tau^* = f^* = e^* = 0$ is an equilibrium. If $FOC_f > 0$ for $\tau = f = e = 0$, then choosing $f^* > 0$ such that $FOC_f = 0$ for $\tau = e = 0$ implies that $FOC_\tau < 0$ and $FOC_e < 0$, consistent with the most-preferred allocation ($f^* > 0$, $\tau^* = e^* = 0$). Alternatively, choosing $e^* > 0$ such that $FOC_e = 0$ implies that $FOC_f > 0$, which is inconsistent with $e^* > 0$ being part of an equilibrium, while a similar reasoning applies to choosing $\tau^* > 0$.
(b) $w < \bar{w}/\pi$ implies that $FOC_\tau > FOC_f$ $\forall(\tau, f, e)$ while $\beta < \bar{w}/\pi$ implies that $FOC_\tau > FOC_e$ $\forall(\tau, f, e)$. The rest of proof is similar to part (a).
(c) $w < \beta$ implies that $FOC_e > FOC_f$ $\forall(\tau, f, e)$ while $\beta > \bar{w}/\pi$ implies that $FOC_e > FOC_\tau$ $\forall(\tau, f, e)$. The rest of proof is similar to part (a).
(d) If $\beta > \bar{w}/\pi$, then $FOC_e > FOC_\tau$ $\forall(\tau, f, e)$ and no child ever most-prefer $\tau^* > 0$. Propositions 1, 2 and 3 have jointly established that the amount of (formal or informal) help is minimum for the child with productivity $w = \beta$. The condition for this child to give a strictly positive amount of help is

$$u'(T\beta) < \alpha\phi'(\bar{p}).$$

If $\beta < \bar{w}/\pi$, then $FOC_\tau > FOC_e$ $\forall(\tau, f, e)$ and no child ever most-prefers $e^* > 0$. In that case, children with $w < \bar{w}/\pi$ most-prefer $\tau^* > 0$ while children with $w > \bar{w}/\pi$ most-prefer $f^* > 0$. Proposition 5(a) will show that τ^* is decreasing in w while Proposition 3 has established that f^* is increasing in w. Hence, the child who gives the lowest amount of aid has a productivity level of $w = \bar{w}/\pi$, and the condition for this child to give a strictly positive amount is

$$u'(T\bar{w}/\pi) < \alpha\phi'(\bar{p}).$$

Proof of Proposition 5

(i) Applying the implicit function theorem on (6) together with $f = e = 0$, we obtain

$$sign\left(\frac{d\tau}{dw}\right) = sign\left(-u'(c) - cu''(c)\right) \leq 0$$

under Assumption 2(a). Part (b) is proved similarly. Part (c) is obtained applying the implicit function theorem together with Assumption 2(b) so that

$$sign\left(\frac{d\tau}{d\pi}\right) = sign\left(-\frac{\alpha\bar{w}}{\pi^2}\phi'(x)\left[1 - \frac{\bar{w}\tau/\pi}{x}\left(-\frac{\phi''(x)x}{\phi'(x)}\right)\right]\right) < 0$$

where $x = \bar{p} + \tau\bar{w}/\pi$.

(ii) Observe that the FOCs for τ when $e = f = 0$ and for e when $\tau = f = 0$ can both be written as

$$-wu'\left(w(T - x)\right) + \alpha y\phi'\left(\bar{p} + yx\right) = 0,$$

where $(x, y) \in \{(\tau, \bar{w}/\pi), (e, \beta)\}$. Applying the implicit function theorem, we obtain that

$$sign\left(\frac{dx}{dy}\right) = sign\left(\alpha\phi'\left(\bar{p} + yx\right) + \alpha yx\phi''\left(\bar{p} + yx\right)\right) > 0$$

by Assumption 2(b). Hence, $\beta < \bar{w}/\pi \Rightarrow e^* < \tau^* \Rightarrow \beta e^* < \tau^*\bar{w}/\pi$.

Proof of Proposition 6

(a) If $w < \beta$, then $f^* = 0$ and $e^* \geq 0$. Assuming $e^* > 0$, the FOC for τ is given by

$$\begin{aligned} \frac{\partial U_O^D}{\partial \tau} &= v_d\left[\frac{\bar{w}}{\pi} + \beta\frac{de^*}{d\tau}\right] \quad (10) \\ &= \frac{\phi'\left(\bar{p} + \tau\bar{w}/\pi + \beta e^*\right)}{w^2u''\left(w(T - \tau - e^*)\right) + \alpha\beta\phi''\left(\bar{p} + \tau\bar{w}/\pi + \beta e^*\right)}w^2u''\left(w(T - \tau - e^*)\right)\left[\frac{\bar{w}}{\pi} - \beta\right] \end{aligned}$$

where we replaced for the expression for $de^*/d\tau$ in (3). We know from Proposition 2(c) that $de^*/d\tau < 0$ so that pushing τ above some threshold results in $e^* = 0$.[20] In that case, we have that

$$\frac{\partial U_O^D}{\partial \tau} = \frac{\bar{w}}{\pi}\phi'\left(\bar{p} + \tau\bar{w}/\pi\right) > 0. \quad (11)$$

When $\beta < \bar{w}/\pi$, then (10)> 0, so that U_O^D is increasing in τ (whether e^* is positive or nil) and $\tau^* = 1$.

(b and c) If $w > \beta$, then $f^* \geq 0$ and $e^* = 0$. Assuming $f^* > 0$, the FOC for τ is given by

$$\begin{aligned} \frac{\partial U_O^D}{\partial \tau} &= v_d\left[\frac{\bar{w}}{\pi} + \frac{df^*}{d\tau}\right] \\ &= \frac{\phi'\left(\bar{p} + \tau\bar{w}/\pi + f^*\right)}{u''\left(w(T - \tau) - f^*\right) + \alpha\phi''\left(\bar{p} + \tau\bar{w}/\pi + f^*\right)}u''\left(w(T - \tau) - f^*\right)\left[\frac{\bar{w}}{\pi} - w\right] \quad (12) \end{aligned}$$

[20] The assumption mentioned in footnote 14 implies that this threshold is lower than 1.

where we replaced for the expression for $df^*/d\tau$ in (4). We know from Proposition 3(c) that $df^*/d\tau < 0$ so that pushing τ above some threshold may result in $f^* = 0$.[21] In that case, we have that $\partial U_O^D/\partial\tau$ is given by (11).

(b) If $w < \bar{w}/\pi$, then (12)> 0, so that U_O^D is increasing in τ (whether f^* is positive or nil) and $\tau^* = 1$.

(c) If $w > \bar{w}/\pi$, then (12)< 0, so that zero is the agent's most-preferred value of τ among the values of τ low enough that $f^*(\tau, w) > 0$. Alternatively, one is the agent's most-preferred value of τ among the values of τ large enough that $f^*(\tau, w) = 0$. Comparing the elderly's utility level with $\tau = 0$ (given by $v(\bar{p} + f^*(0, w))$) and with $\tau = 1$ (given by $v(\bar{p} + \bar{w}/\pi)$), we obtain that the elderly prefers $\tau = 0$ if

$$f^*(0, w) > \frac{\bar{w}}{\pi}.$$

Since we know from Proposition 3(b) that $f^*(0, w)$ is increasing in w, we define $\breve{w}$ in the following way: (i) If $f^*(0, w_+) > \bar{w}/\pi$ then $\breve{w}$ is such that $f^*(0, \breve{w}) = \bar{w}/\pi$,[22] (ii) If $f^*(0, w_+) < \bar{w}/\pi$ then $\breve{w} = w_+$.

Proof of Proposition 8

(a) We have already established that preferences of dependent parents with $w > \bar{w}/\pi$ are not single-peaked (see proposition 6 (c)), so that we cannot apply the usual median voter theorem. De Donder (2013) shows that, if voters can be grouped into exogenous categories (i.e., here, categories not depending on τ), and if the single-crossing property à la Gans and Smart (1996) is satisfied inside each group of voters, then a global Condorcet winning value of τ exists. We then form four exogenous groups of voters, according to their age (child or parent) and to whether the child in the family prefers informal to formal help ($w < \beta$) or the opposite ($w > \beta$).

(i) We first show that preferences of young agents with dependent parents are indeed single-crossing. Their indirect utility function as a function of τ is

$$V_Y^D = u(w(T - \tau - e^* - f^*)) + \alpha\phi(\bar{p} + b + \beta e^* + f^*)$$

where $e^* = e^*(\tau, w)$ and $f^* = f^*(\tau, w)$. Using the envelope theorem, the marginal rate of

[21] The assumption mentioned in footnote 14 implies that $f^*(\tau, w) = 0$ for $\tau > \tilde{\tau}$ with $\tilde{\tau} < 1$.

[22] It is immediate that $f^*(0, \bar{w}/\pi) < \bar{w}/\pi$.

substitution between b and τ is

$$MRS = -\frac{\partial V_Y^D/\partial \tau}{\partial V_Y^D/\partial b} = \frac{wu'(c)}{\alpha\phi'(x)}$$

where $c = w(T-\tau-e^*)$ and $x = b+\bar{p}+\beta e^*$ for $w < \beta$, and $c = w(T-\tau)-f^*$ and $x = b+\bar{p}+f^*$ when $w > \beta$.

If $w < \beta$, then $f^* = 0$ and $e^* \geq 0$. If $e^* > 0$, then using the FOC for e (equation (2)), we obtain that $MRS = \beta$. If $e^* = 0$, we make use of the fact that the same FOC for e^* is negative for $e = 0$ to obtain that the MRS is larger than β and is weakly increasing in w given Assumption 2. Observe that the government budget constraint $b(\tau)$ is linear in τ with a slope $\bar{w}/\pi$. Tangency between indifference curve and budget constraint then occurs on the strictly concave part of the indifference curve, where $e^* = 0$ for the individual (see Proposition 4(b)). Preferences are single-crossing over these parts (see Epple and Romano (1996, Figure 4)).

If $w > \beta$, then $e^* = 0$ and $f^* \geq 0$. If $f^* > 0$, then using the FOC for f (equation (1)), we obtain that $MRS = w$. If $f^* = 0$, we make use of the fact that the same FOC for f^* is negative for $f = 0$ to obtain that the MRS is larger than w and is weakly increasing in w given Assumption 2. The strictly concave parts of the indifference curves are then single-crossing (agents with $w > \bar{w}/\pi$ most prefer $\tau^* = 0$).

We now study the preferences of dependent individuals. The indirect utility function of a dependent elderly whose child has $w < \beta$ is $V_O^D = \phi(\bar{p} + b + \beta e^*)$, which is monotonically increasing in τ for all values of $w < \beta$ (see the proof Proposition 6(a)). The single-crossing condition (see Gans and Smart, 1996, section 2.1) is then trivially satisfied, since all agents with $w < \beta$ prefer any larger to any smaller value of τ (graphically, indifference curves all cut the budget constraint from above in the (τ, b) space, since $\beta \mid de^*/d\tau \mid < \bar{w}/\pi$ as shown in the proof of Proposition 6(a)).

The indirect utility function of a dependent elderly whose child has $w > \beta$ is $V_O^D = \phi(\bar{p} + b + f^*)$ so that the MRS between τ and b is equal to $MRS = -df^*/d\tau$ and $dMRS/dw = -d^2 f^*/d\tau dw$.

So, if $d^2 f^*/d\tau dw < 0$, the preferences of the four exogenous sets of voters are single-crossing and, by De Donder (2013), the global Condorcet winner exists and corresponds to the median most-preferred value of τ.

(ii) We show that logarithmic preferences satisfy the single crossing property for dependent parents with $w > \beta$. Observe that

$$\frac{d^2 f^*}{d\tau dw} = \frac{-1}{[u''(c) + \alpha\phi''(x)]^2} \left[u''(c)^2 + \alpha\phi''(x)u''(c) + \alpha(w - \frac{\bar{w}}{\pi})(\frac{dc}{dw}\phi''(x)u'''(c) - \frac{dx}{dw}\phi'''(x)u''(c)) \right]$$

which is obtained from (4) after tedious computations. In the case of a logarithmic utility, the expression inside brackets becomes

$$\frac{1}{c^4} + \frac{\alpha}{c^2 x^2} + (w - \frac{\bar{w}}{\pi})\alpha \left[\frac{-2}{x^2 c^3}\frac{dc}{dw} + \frac{2}{x^3 c^2}\frac{dx}{dw} \right]. \tag{13}$$

Fully differentiating (1) with respect to w, we obtain

$$\frac{dc}{dw} = \frac{\alpha c^2}{x^2}\frac{dx}{dw}.$$

Replacing this expression into (13), we obtain

$$\frac{1}{c^4} + \frac{\alpha}{c^2 x^2} + (w - \frac{\bar{w}}{\pi})\alpha \frac{dx}{dw}\frac{2}{x^3 c} \left[\frac{1}{c} - \frac{\alpha}{x} \right]$$

with $[\frac{1}{c} - \frac{\alpha}{x}] = 0$ from the FOC with respect to f, (1).

Hence

$$\frac{d^2 f^*}{d\tau dw} = \frac{-1}{[u''(c) + \alpha\phi''(x)]^2} \left[\frac{1}{c^4} + \frac{\alpha}{c^2 x^2} \right] < 0.$$

(iii) Since condition (7) is not satisfied, we have that

$$\pi \left(F(\bar{w}/\pi) + F(\breve{w}) \right) > \frac{1+\pi}{2},$$

where the LHS represents the maesure of children with a dependent parent and of old dependents who most prefer a strictly positive value of τ. At the same time, we have that

$$\pi F(\breve{w}) < \frac{1+\pi}{2},$$

since $F(\breve{w}) < 1$ and $\pi \leq (1+\pi)/2 \; \forall \pi \leq 1$. Thus, old dependents who prefer the maximum tax rate cannot form a majority by themselves. The decisive voter is therefore a young agent with a dependent parent and with a productivity level, denoted by w^{GCW}, such that $w^{GCW} < \bar{w}/\pi$. Voters who want a larger value of τ are young agents with dependent parents and $w < w^{GCW}$ together with old dependent agents with $w < \breve{w}$ (the other groups of agents want a lower value of τ) and must represent one half of the voters. This is how we implicitly define w^{GCW}:

$$\pi \left(F(\breve{w}) + F(w^{GCW}) \right) = \frac{1+\pi}{2}.$$

(b) Condition $f^*(\tau^{LCW}, w_+) = 0$ implies that $f^*(\tau^{LCW}, w) = 0$ for every child of a dependent parent with $\bar{w}/\pi < w < w_+$, since $f^*(\tau, w)$ is increasing in w, $\forall \tau$. Then, every dependent parent with $w > \bar{w}/\pi$ has locally increasing utilities around τ^{LCW} as $db/d\tau > 0$ (see equation (11)). We already know that the utility of all dependent agents with $w < \bar{w}/\pi$ monotonically increases with τ (see the proof of Proposition 6(a) and (b)). The dependent parents form a mass of measure π. As for children with dependent parents, we know from Proposition 5 (a) that their most-preferred value of τ is decreasing with w, so that all of them with $w < w^{LCW}$ prefer a larger-than-τ^{LCW} value of τ. The mass $\pi\left(1 + F(w^{LCW})\right)$ then represents the mass of voters who prefer a slightly larger value of τ than τ^{LCW}, so that τ^{LCW} is a local Condorcet winner if this mass represents exactly one half of voters.

(c) Result obtained after long and tedious but straightforward manipulations of the FOCs for formal and informal help (equations (1) and (2)), available from the authors.