Dynastic Precautionary Savings

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Facing Demographic Change in a Challenging Economic Environment
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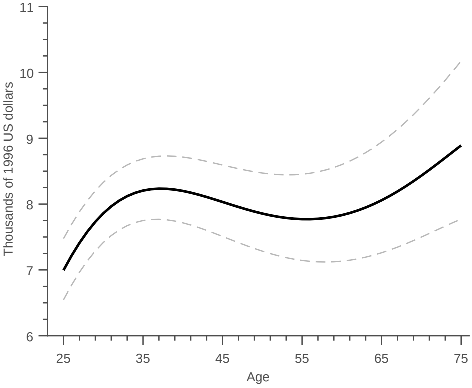
Motivation

- Consumption of retired parents is backloaded
- Backloading postdates the resolution of own income risk



Details   Non-parents

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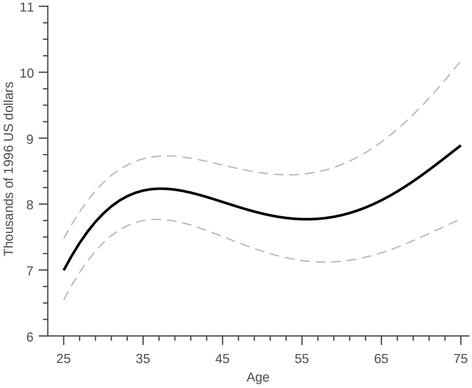

Possible reasons:

- Health expenditure risk
- Transfers from children

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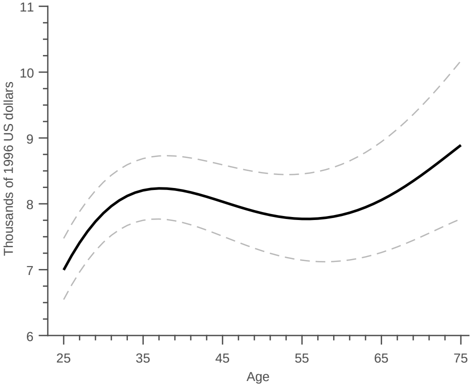


Possible reasons:

- Health expenditure risk
- Transfers from children

This paper:

- Resolution of children's income risk

$\Downarrow$

dynastic precautionary savings

Details   Non-parents

Contributions of this paper

1. Provide empirical evidence for dynastic precautionary behavior
   - Examine the response of parent's consumption to child's income risk
   - Exploit variation in permanent income risk across age and sectors
   - Analyze robustness to endogeneity concerns
2. Build a model of dynastic precautionary saving
   - Parent and child save separately: non-cooperative + no commitment
     - Can identify wealth position of overlapping generations + size and timing of intergenerational transfers
     - Strategic interactions between parent and child
     - Contrast with unitary household model (no strategic interactions)
   - Counterfactual
     - Contribution to parental wealth and intergenerational transfers

Preview of results

Empirical

- Parent's consumption decreases with child's permanent income risk
  - Response is nearly as large as to own income risk
- Permanent income risk is decreasing over age, with variation across sectors (both in levels and slopes)
  - Parents of children younger than 40 consume $2,945 less per year because uncertainty is yet to be resolved (conditional on controls)
  - Parents of children in finance sector consume 3% less than parents of government employees because of higher uncertainty (conditional on controls)

Preview of results

Quantitative

- Model with strategic interactions predicts dynastic precautionary behavior closer to data than model without strategic interactions
  - *No strategic interactions*: dynastic precautionary motive is more important than precautionary motive
  - *Strategic interactions*: relative importance of precautionary motives is flipped because of overconsumption by children
- Counterfactual
  - Dynastic precautionary wealth is $\approx \frac{1}{4}$ of aggregate wealth
  - Intergenerational transfers are mostly driven by dynastic uncertainty

Related literature

Consumption-saving over the life-cycle, especially at older age

- *mortality and medical risk*: Hubbard et al. (1995), Palumbo (1999), de Nardi et al. (2010), Kopecky and Koreshova (2014)
- *bequest motive*: Kotlikoff and Summers (1981), Kopczuk and Lupton (2007), Ameriks et al. (2011), de Nardi et al. (2013), Lockwood (2013)

Precautionary savings

- Kimball (1990), Strawczynski (1994), Caroll and Samwick (1997), Gourinchas and Parker (2002), Cagetti (2003), Kennickell and Lusardi (2005), Hurst et al. (2010)

Family as insurance

- *empirical studies*: Altonji et al. (1996), Attanasio et al. (2015), McGarry (1999, 2016)
- *dynamics models of families*: Nishiyama (2002), Kaplan (2012), Barczyck and Kredler (2014, 2016), Fahle (2015), Mommaerts (2015), Ameriks et al. (2016), Luo (2016)

Outline

1. Empirical analysis
   - Data description
   - Income uncertainty
   - Test for dynastic precautionary savings
   - Robustness analysis
2. Model
   - Environment and parameter values
   - Comparison between models
   - Counterfactual
3. Conclusion

Empirical evidence

Empirical test

- Pure life-cycle models (including warm-glow altruism) imply:

$$c_p = F_p\left(Y_p, \sigma_p; \mathbf{X_p}\right) \quad \text{and} \quad c_c = F_c\left(Y_c, \sigma_c; \mathbf{X_c}\right)$$

- Models with altruism à la Barro (1974) imply:

$$c_p = \bar{F}_p\left(Y_p, \sigma_p, Y_c, \sigma_c; \mathbf{X_p}, \mathbf{X_c}\right) \quad \text{and} \quad c_c = \bar{F}_c\left(Y_c, \sigma_c, Y_p, \sigma_p; \mathbf{X_p}, \mathbf{X_c}\right)$$

Test by regressing:

- c_p on parent's income uncertainty and child's income uncertainty
- c_c on parent's income uncertainty and child's income uncertainty

Data

- Parent-child pairs
  - PSID Family Identification Mapping System
  - Parent with n children $\Rightarrow$ n parent-child pairs
- Income uncertainty
  - PSID 1968-2013
  - Stratify by age and sector (N occupations $\times$ M industries)
- Consumption
  - Later years (2005-2013): consumption directly from PSID
  - Early waves (1981-2003): use CEX to impute consumption based on an inverted food demand equation (Blundell et al., 2008)

Income uncertainty

- Income uncertainty about future income stream (permanent income)

$$Y_h^i \equiv \sum_{j=h+1}^{H} \frac{y_j^i}{R^{j-h}}$$

- Treat uncertainty as the standard deviation of forecast error of Y_h^i

- Predicted permanent income as of age h is

$$\hat{Y}_h^i \equiv \sum_{j=h+1}^{H} \frac{\hat{y}_{j,h}^i}{R^{j-h}}$$

Income uncertainty

How are earnings predicted?

$$y_j^i = \underbrace{\theta_0 + \mathbf{X}_h^i \boldsymbol{\theta}_1 + \theta_3 \boldsymbol{t}_j}_{\hat{y}_{j,h}} + e_{j,h}^i$$

$\mathbf{X}_h^i$: current and lagged income, age polynomial, dummies for current educational attainment, marital status, race and family size

$\boldsymbol{t}_j$: time trend

Income uncertainty

How are earnings predicted?

forecast error of age $j > h$ income

$$y_j^i = \underbrace{\theta_0 + \mathbf{X}_h^i \boldsymbol{\theta}_1 + \theta_3 \boldsymbol{t}_j}_{\hat{y}_{j,h}} + e_{j,h}^i$$

$\mathbf{X}_h^i$: current and lagged income, age polynomial, dummies for current educational attainment, marital status, race and family size

$\boldsymbol{t}_j$: time trend

Income uncertainty

- The forecast error of permanent income is

$$\mathcal{E}_h^i \equiv \sum_{j=h+1}^{H} \frac{e_{j,h}^i}{R^{j-h}}$$

where $e_{j,h}^i = y_j - \hat{y}_{j,h}^i$.

- Permanent income uncertainty

$$\mathrm{Std}_i\left(\mathcal{E}_h^i\right) = \mathrm{Std}_i\left(\sum_{j=h+1}^{H} \frac{e_{j,h}^i}{R^{j-h}}\right)$$

Income uncertainty

- The forecast error of permanent income is

$$\mathcal{E}_h^i \equiv \sum_{j=h+1}^{H} \frac{e_{j,h}^i}{R^{j-h}}$$

where $e_{j,h}^i = y_j - \hat{y}_{j,h}^i$.

- Permanent income uncertainty

$$\mathrm{Std}_i\left(\mathcal{E}_h^i\right) = \mathrm{Std}_i\left(\sum_{j=h+1}^{H} \frac{e_{j,h}^i}{R^{j-h}}\right)$$

- Stratify individuals by sector s:

$$\mathrm{Std}_s\left(\mathcal{E}_h^i\right) = \left(\sum_{j=h+1}^{H} \frac{\mathrm{Var}_s\left(e_{j,h}^i\right)}{R^{2(j-h)}} + 2\sum_{j=h+1}^{H-1} \frac{1}{R^{j-h}} \sum_{k=j+1}^{H} \frac{\mathrm{Cov}_s\left(e_{j,h}^i; e_{k,h}^i\right)}{R^{k-h}}\right)^{\frac{1}{2}}$$

▸ Measurement error ▸ Attrition

Income uncertainty over age ▸ Flow

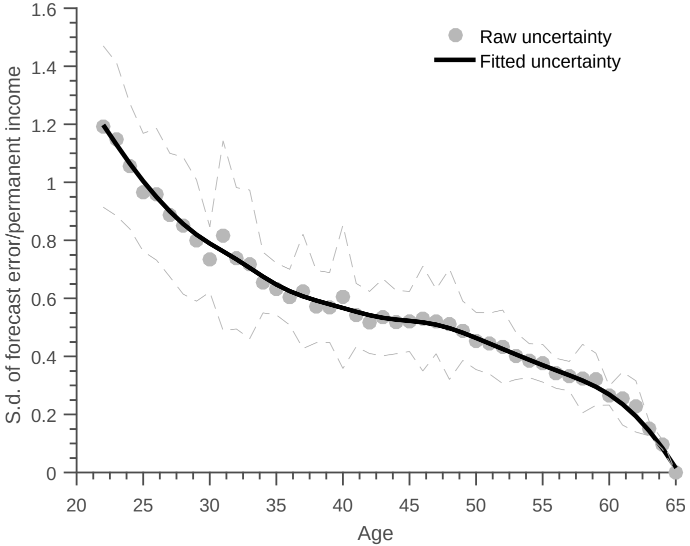


Figure: Age Profile of Income Uncertainty (Relative to Permanent Income)

Income uncertainty over age and sectors

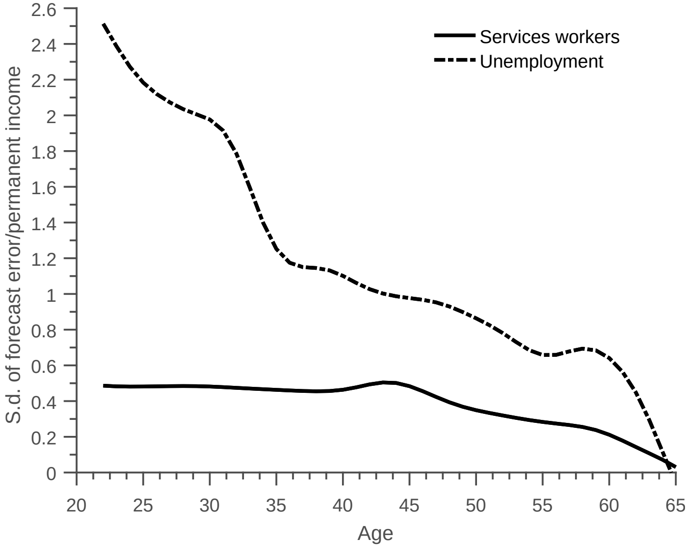


Figure: Age Profile of Income Uncertainty (Relative to Permanent Income)

Empirical specification

$$
\begin{aligned}
\ln c_p &= \beta_0^p + \beta_1^p \sigma_p + \beta_2^p \sigma_c + \mathbf{X}_p \beta_3^p + \mathbf{X}_c \beta_4^p + \epsilon_p \\
\ln c_c &= \beta_0^c + \beta_1^c \sigma_p + \beta_2^c \sigma_c + \mathbf{X}_p \beta_3^c + \mathbf{X}_c \beta_4^c + \epsilon_c
\end{aligned}
$$

c_p, c_c: consumption of parent and child

σ_p: parent's permanent income uncertainty

σ_c: child's permanent income uncertainty

$\mathbf{X_p}, \mathbf{X_c}$: full set of age dummies; dummies for marital status, race, gender, educational attainment, family size; permanent income, wealth holdings

Results

Table: Regression of Consumption (non-durables and services) on Income Risk

	Parent's consumption	Child's consumption
Parent's uncertainty	-0.089** (0.033)	-0.039 (0.025)
Child's uncertainty	-0.081* (0.034)	-0.163** (0.038)

Note: Bootstrapped robust std errors clustered at parent level in parentheses; $^{*}p < 5\%$; $^{**}p < 1\%$

- Parents of children younger than 40 consume, on average, $2,945 less per year because most of dynastic uncertainty is to be resolved
- Parents of construction workers consume, on average, 2.5% less than parents of services workers because of the uncertainty differential

Full table

Robustness analysis

- Endogeneity concerns
  1. Health risk: include health controls ▸ Health
  2. Selection into risky sectors:
     - prob. of moving to high risk sector is not lower if parent looses job
     - control for initial sector ▸ Sector

- Also robust to
  1. Heterogeneous bequest motives ▸ Bequest
  2. Information set used to predict income ▸ Other
  3. Time and geography dummies

Model

Objective

Can we write a model that predicts dynastic precautionary saving behavior consistent with the data?

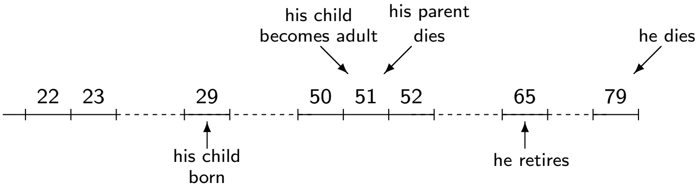
- Model with strategic interactions between parents and children
- Contrast with unitary household model (no strategic interactions)

What are the implications of dynastic precautionary saving for:

- inter-vivos transfers and bequest
- parental wealth?

Environment

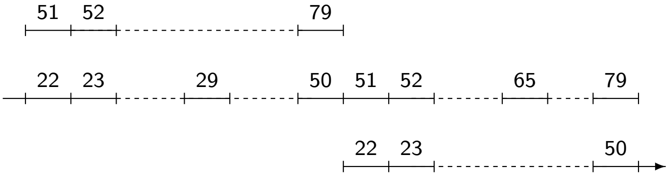
- Life-cycle of an individual



- Work in sector s until retirement and earn risky labor income: y_p, y_c
- No income risk after retirement: $\Phi\left(\hat{y}_p\right)$
- Pay proportional tax τ on labor income
- Hold government bond with gross return R: a_p, a_c

Environment

- Overlapping generations



- Parent-child pairs indexed by age: (h_p, h_c)
- Intergenerational altruism: parent places weight γ on child's utility
  $\rightarrow$ makes inter-vivos transfers g_p and end-of-life bequest
- Parent and child overlap for 29 years

Model with strategic interactions: Timing

- Each year they overlap, parent and child play a 2-stage game

  1. Stage 1. Parent chooses consumption c_p, wealth a'_p and transfers g_p

     State variable: $\tilde{s}_p = (a_p, a_c, y_p, y_c, s_p, s_c)$

  2. Stage 2. Child decides consumption c_c and wealth a'_c

     State variable: $\tilde{s}_c = (a_c, y_c, y_p, g_p, a'_p, s_p, s_c)$

- Equilibrium concept is MPE

- Solve backwards

- Can identify wealth position of overlapping generations, as well as size and timing of intergenerational transfers

Decision problems

Model without strategic interactions

- Setup
  - While alive, parent makes all consumption-saving decisions
  - Family budget constraint: $c_p + c_c + a' = (1 - \tau)(y_p + y_c) + Ra$
- Wealth position of parent and child cannot be separately identified
- Size and timing of intergenerational transfers is indeterminate

▸ Decision problems ▸ Government

Parameter values

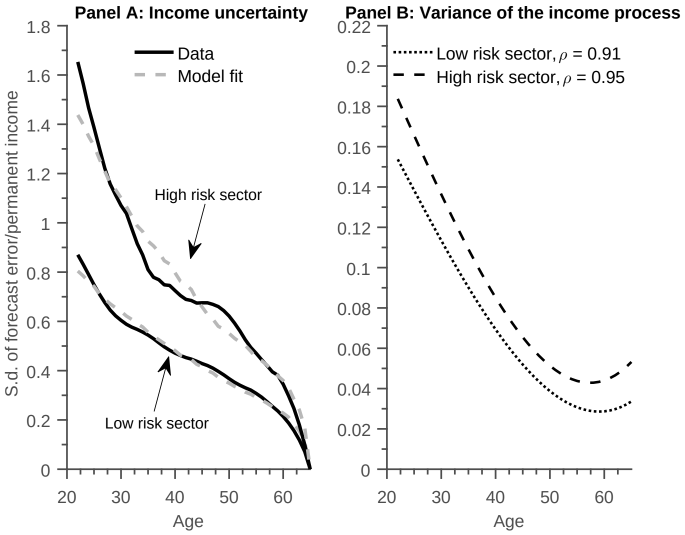
- Two sectors: low risk and high risk
  $\rightarrow$ group the 17 empirical sectors based on whether risk is below/above average
- Exogenous transition between sectors (including intergenerational)

$$\mathbf{P_s} = \begin{bmatrix} 0.921 & 0.079 \\ 0.113 & 0.887 \end{bmatrix} \quad \text{and} \quad \mathbf{P_s^{ig}} = \begin{bmatrix} 0.647 & 0.353 \\ 0.493 & 0.507 \end{bmatrix}$$

- Income process

$$\ln y_{hs}^i = f(h) + \tilde{y}_{hs}^i \quad \text{and} \quad \tilde{y}_{hs}^i = \rho_s \tilde{y}_{h-1,s}^i + \epsilon_{hs}^i,\ \epsilon_{hs} \sim \left(0, \sigma_{hs}^2\right)$$

Parameter values



Parameter values

Parameter	Value	Justification/Target
a, b	0.168, 0.355	$\Phi(\hat{y}) = a\bar{y} + b\hat{y}$, Guvenen et al. (2013)
σ	2	Standard
β	0.959/0.958	Wealth to income ratio
γ	0.201/0.71	Parent-child consumption ratio
τ	0.246	US average tax rate (OECD Tax Database)
R	1.04	Initial steady-state, G set accordingly
$\underline{A}_h$	0	Sensitivity analysis to negative borrowing limit

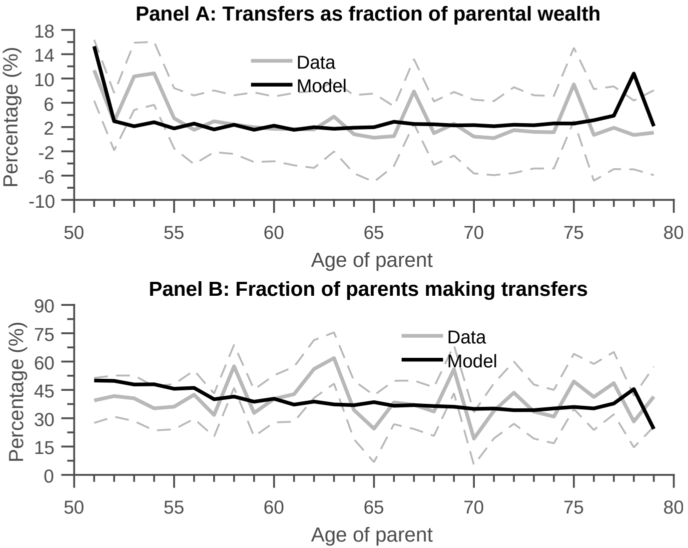
Table: Parameter Values

Model predictions

Table: Regression of Consumption on Income Risk (Models vs Data)

	Model without strategic interactions	Model with strategic interactions	Data
Parent's uncertainty	-0.022**	-0.098**	-0.089** $[-0.153 \quad -0.025]$
Child's uncertainty	-0.062**	-0.067**	-0.081* $[-0.147 \quad -0.015]$

Model predictions: inter-vivos transfers



▸ Wealth Distribution

Counterfactual: Contribution to wealth and transfers

Implementation:

1. Shut down income risk of children (individuals of age 22-50)
   - evaluate effect on intergenerational transfers
   - not suited to evaluate effect on wealth accumulation
2. Two-step approach
   - shut down all income risk $\Rightarrow$ recover precautionary and dynastic precautionary wealth
   - solve life-cycle model with and without risk $\Rightarrow$ recover precautionary wealth
   - difference is dynastic precautionary wealth

Counterfactual: Results

Table: The effect of eliminating dynastic uncertainty

	Aggregate Wealth	Intergenerational Transfers		
		Total	Inter-vivos transfers	End-of-life bequest
Total effect (%):	-27.37	-97.48	-99.82	-90.80

Counterfactual: Results

Table: The effect of eliminating dynastic uncertainty

	Aggregate Wealth	Intergenerational Transfers		
		Total	Inter-vivos transfers	End-of-life bequest
Total effect (%):	-27.37	-97.48	-99.82	-90.80

Caveats:

- crowding out between wealth components
- missing saving motives relevant at old age

How much consumption insurance via DPS?

- *Consumption insurance coefficient* in dynastic vs life-cycle model

$$\phi^{\epsilon} = 1 - \frac{\text{Cov}\left(\Delta c_{ih}, \epsilon_{ih}\right)}{\text{Var}\left(\epsilon_{ih}\right)}$$

- Parent's dynastic precautionary saving accounts for 26% of the total consumption insurance of children

- The benefit is largest for children in high-risk sector

Conclusion

- Consumption of retired parents is backloaded
- This is largely a reflection of dynastic precautionary saving
- Implications:
  - Precautionary savings across generations $\Rightarrow$ infinite horizon model
  - Design of social insurance policies: guaranteed minimum income, unemployment insurance
- Dynastic precautionary savings might help explain other facts
  - Retirees deplete wealth slower than the life-cycle model predicts
  - There is substantial wealth heterogeneity at retirement, even after controlling for realized lifetime income

Age profile of consumption

$$\ln C_{it} = \beta_0 + \boldsymbol{\beta}_{age} f\left(Age_{it}\right) + \boldsymbol{\beta}_c Coh_i + \beta_t D_t + \boldsymbol{\beta}_x \mathbf{X}_{it} + \varepsilon_{it}$$

C_{it}: consumption expenditure

$f\left(Age_{it}\right)$: quartic polynomial in age

Coh_i: 10-year cohort dummies

D_t: year dummies

$\mathbf{X_{it}}$: dummies for race, educational attainment, family size and employment

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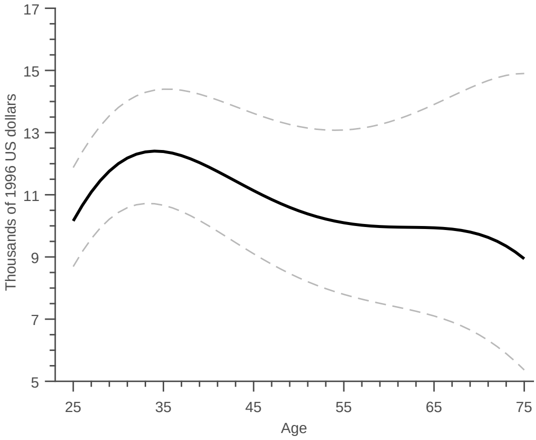
Age profile of consumption: non-parents



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Measurement error

If measurement error is:

- iid across sectors with variance $\sigma_{0,h}^2$
- uncorrelated with the true forecast error

then measured income uncertainty $\tilde{\mathrm{Var}}_s\left(\mathcal{E}_h^i\right)$ is

$$\tilde{\mathrm{Var}}_s\left(\mathcal{E}_h^i\right) = \underbrace{\mathrm{Var}_s\left(\mathcal{E}_h^i\right)}_{\text{true income risk}} + \underbrace{\sum_{j=h+1}^{H} \frac{\sigma_{0,h}^2}{R^{2(j-h)}}}_{\text{measurement error}}$$

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Sample attrition - example

	A			B			C		
Period 1	$e^A_{1,1}$	$e^A_{2,1}$	$e^A_{3,1}$	$e^B_{1,1}$	$e^B_{2,1}$	$e^B_{3,1}$	$e^C_{1,1}$	$e^C_{2,1}$	$e^C_{3,1}$
Period 2		$e^A_{2,2}$	$e^A_{3,2}$		$e^B_{2,2}$	$e^B_{3,2}$		$e^C_{2,2}$	$e^C_{3,2}$
Period 3			$e^A_{3,3}$			$e^B_{3,3}$			$e^C_{3,3}$

$$\mathrm{Std}_s\left(\mathcal{E}^i_1\right) = \left(\frac{\mathrm{Var}_s\left(e^i_{2,1}\right)}{R^2} + \frac{\mathrm{Var}_s\left(e^i_{3,1}\right)}{R^4} + 2\frac{\mathrm{Cov}_s\left(e^i_{2,1}; e^i_{3,1}\right)}{R \times R^2}\right)^{\frac{1}{2}}$$

where $\mathrm{Var}_s\left(e^i_{3,1}\right) = \frac{\left(e^A_{3,1}\right)^2 + \left(e^B_{3,1}\right)^2 + \left(e^C_{3,1}\right)^2}{2}$

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Sample attrition - example

	A			B			C		
Period 1	$e^A_{1,1}$	$e^A_{2,1}$	$e^A_{3,1}$	$e^B_{1,1}$	$e^B_{2,1}$	$e^B_{3,1}$	$e^C_{1,1}$	$e^C_{2,1}$	~~$e^C_{3,1}$~~
Period 2		$e^A_{2,2}$	$e^A_{3,2}$		$e^B_{2,2}$	$e^B_{3,2}$		$e^C_{2,2}$	~~$e^C_{3,2}$~~
Period 3			$e^A_{3,3}$			$e^B_{3,3}$			~~$e^C_{3,3}$~~

$$\mathrm{Std}_s\left(\mathcal{E}^i_1\right) = \left(\frac{\mathrm{Var}_s\left(e^i_{2,1}\right)}{R^2} + \frac{\mathrm{Var}_s\left(e^i_{3,1}\right)}{R^4} + 2\frac{\mathrm{Cov}_s\left(e^i_{2,1}; e^i_{3,1}\right)}{R \times R^2}\right)^{\frac{1}{2}}$$

where $\mathrm{Var}_s\left(e^i_{3,1}\right) = \frac{\left(e^A_{3,1}\right)^2 + \left(e^B_{3,1}\right)^2 + \cancel{\left(e^C_{3,1}\right)^2}}{2}$

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Flow Uncertainty

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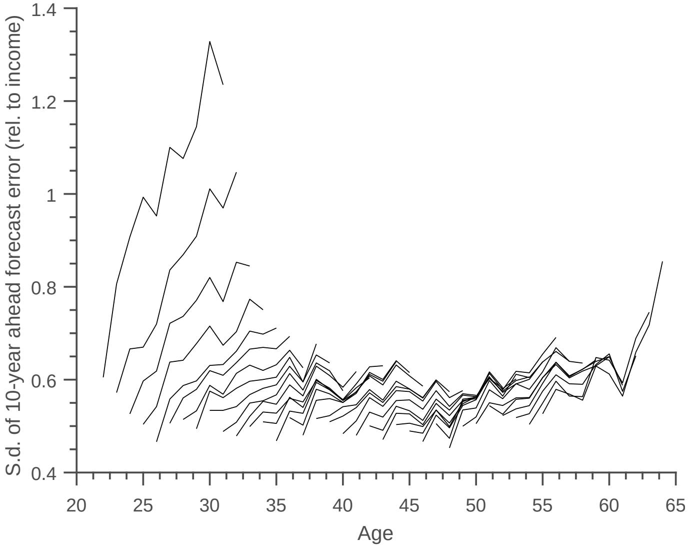

Figure: Relative Std Dev of the 10-year-ahead Earnings Forecasts

Estimation results

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	Non-durable consumption		Total consumption	
	Parent's consumption	Child's consumption	Parent's consumption	Child's consumption
Parent's uncertainty	-0.089** (0.033)	-0.039 (0.025)	-0.081** (0.030)	-0.043 (0.025)
Child's uncertainty	-0.081* (0.034)	-0.163** (0.038)	-0.076* (0.033)	-0.149** (0.038)
$\mathbf{X}_p$				
Marital status	0.246** (0.057)	-0.024 (0.047)	0.251** (0.058)	-0.039 (0.046)
Race	0.132** (0.049)	-0.017 (0.056)	0.132** (0.049)	-0.026 (0.056)
Educ: some college	0.247** (0.030)	0.150** (0.026)	0.247** (0.030)	0.159** (0.026)
Educ: college degree	0.271** (0.024)	0.066** (0.021)	0.271** (0.024)	0.076** (0.021)
Permanent income	0.114** (0.011)	0.063** (0.010)	0.114** (0.013)	0.061** (0.010)
Asset holdings	0.036** (0.003)	0.012** (0.002)	0.036** (0.003)	0.012** (0.002)
$\mathbf{X}_c$				
Marital status	-0.053* (0.023)	0.173** (0.028)	-0.066** (0.023)	0.177** (0.028)
Gender	-0.019 (0.023)	0.288** (0.030)	-0.019 (0.022)	0.296** (0.030)
Educ: some college	0.092** (0.021)	0.093** (0.025)	0.091** (0.021)	0.095** (0.025)
Educ: college degree	0.164** (0.023)	0.171** (0.022)	0.164** (0.021)	0.172** (0.022)
Permanent income	0.014* (0.006)	0.068** (0.006)	0.014* (0.006)	0.066** (0.006)
Asset holdings	0.011** (0.004)	0.049** (0.006)	0.011** (0.004)	0.047** (0.006)
Constant	10.225** (0.413)	11.469** (0.464)	9.833** (0.404)	11.468** (0.463)
R^2	0.288	0.268	0.284	0.276
Sample size	8,851	8,330	8,861	8,323

Health controls

Table: Regression of Parental Consumption on Income Uncertainty

	Baseline	Health Controls
Parent's uncertainty	-0.089** (0.033)	-0.079** (0.029)
Child's uncertainty	-0.081* (0.034)	-0.068 (0.035)

Note: Bootstrapped robust std errors clustered at parent level in parentheses; $^{*}p < 5\%$; $^{**}p < 1\%$

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Initial sector

Table: Regression of Parental Consumption on Income Uncertainty

	Baseline	Initial Sector
Parent's uncertainty	-0.089** (0.033)	-0.083** (0.029)
Child's uncertainty	-0.081* (0.034)	-0.065 (0.035)

Note: Bootstrapped robust std errors clustered at parent level in parentheses; $^{*}p < 5\%$; $^{**}p < 1\%$

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Heterogeneous bequest motives

Table: Regression of Parental Consumption on Income Uncertainty

	Coefficient on parent's risk	Coefficient on child's risk
1. Baseline	-0.089** (0.033)	-0.081* (0.034)
2. Bequest proxy: parent vs non-parent	-0.098** (0.032)	-0.082* (0.033)
3. Bequest proxy: number of children	-0.075 (0.040)	-0.081* (0.034)
4. How important it is leaving an estate?	-0.089** (0.035)	-0.083* (0.034)

Note: Bootstrapped robust std errors clustered at parent level in parentheses; $^{*}p < 5\%$; $^{**}p < 1\%$

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Other robustness tests

Table: Regression of Parental Consumption on Income Uncertainty

	Coefficient on parent's risk	Coefficient on child's risk
1. Baseline	-0.089** (0.033)	-0.081* (0.033)
2. Effect on food consumption	-0.041 (0.022)	-0.009 (0.025)
3. Consumption in later years	-0.139** (0.043)	-0.022 (0.039)
4. Parents with one child	-0.047 (0.055)	-0.136* (0.057)
5. Income forecast with rich information set	-0.075** (0.029)	-0.075* (0.036)
6. Time and geography	-0.070* (0.031)	-0.074* (0.033)

Note: Bootstrapped robust std errors clustered at parent level in parentheses; $^{*}p < 5\%$; $^{**}p < 1\%$

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Decision problems: non-terminal parent

Child's problem:

$$V^c_{h_c}(\tilde{s}_c) = \max_{c_c, a'_c} u(c_c) + \beta \mathbb{E} V^c_{h_c+1}(\tilde{s}'_c | \mathbf{y}, \mathbf{s})$$

$$\text{s.t.} \quad c_c + a'_c = (1-\tau) y_c + R a_c + g_p; \; a'_c \geq A_{h_c}$$

where $\tilde{s}'_c = \left(a'_c, y'_c, y'_p, g'^{\star}_p, a''^{\star}_p, s'_p, s'_c\right)$, $\mathbf{y} = (y_p, y_c)$, $\mathbf{s} = (s_p, s_c)$.

Decision problems: non-terminal parent

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$$V^c_{h_c}(\tilde{s}_c) = \max_{c_c, a'_c} u(c_c) + \beta \mathbb{E} V^c_{h_c+1}(\tilde{s}'_c | \mathbf{y}, \mathbf{s})$$

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Parent's problem:

$$V^p_{h_p}(\tilde{s}_p) = \max_{c_p, a'_p, g_p} u(c_p) + \gamma u\left(c^{\star}_c(\tilde{s}_c)\right) + \beta \mathbb{E} V^p_{h_p+1}\left(\tilde{s}'_p\right)$$

$$\text{s.t.} \quad c_p + a'_p + g_p = (1-\tau) y_p + R a_p; \; a'_p \geq A_{h_p}, \; g_p \geq 0$$

where $\tilde{s}'_p = \left(a'_p, a'^{\star}_c(\tilde{s}_c), y'_p, y'_c, s'_p, s'_c\right)$.

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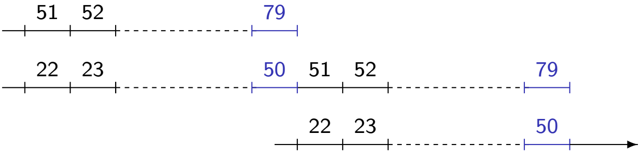
Decision problems: terminal parent



Child's problem:

$$V_{50}^{c}\left(\tilde{s}_c\right) = \max_{c_c, a_c'} u\left(c_c\right) + \beta \mathbb{E} V_{51}^{p}\left(\tilde{s}_p' | \mathbf{y}, \mathbf{s}\right)$$

where $\tilde{s}_p' = \left(a_c' + a_p', 0, y_p', y_c', s_p', s_c'\right)$.

Parent's problem:

$$V_{79}^{p}\left(\tilde{s}_p\right) = \max_{c_p, a_p', g_p} u\left(c_p\right) + \gamma u\left(c_c^{\star}\left(\tilde{s}_c\right)\right) + \beta\gamma \mathbb{E} V_{51}^{p}\left(\tilde{s}_p' | \mathbf{y}, \mathbf{s}\right)$$

where $\tilde{s}_p' = \left(a_c'^{\star}\left(\tilde{s}_c\right) + a_p', 0, y_p', y_c', s_p', s_c'\right)$.

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Decision problems

Non-terminal parent:

$$
\begin{aligned}
V_{h_p}^{p}\left(\tilde{s}_p\right) \quad &= \quad \max_{c_p, c_c, a'} u\left(c_p\right) + \gamma u\left(c_c\right) + \beta \mathbb{E} V_{h_p+1}^{p}\left(\tilde{s}' | \mathbf{y}, \mathbf{s}\right) \\
\text{s.t.} \quad & c_p + c_c + a' = (1-\tau)\left(y_p + y_c\right) + Ra \\
& a' \geq \underline{A}_{h_p} \geq 0
\end{aligned}
$$

where $\tilde{s}' = \left(a', y_p', y_c', s_p', s_c'\right)$.

Terminal parent:

$$
\begin{aligned}
V_{79}^{p}\left(\tilde{s}_p\right) \quad &= \quad \max_{c_p, c_c, a'} u\left(c_p\right) + \gamma u\left(c_c\right) + \beta \gamma \mathbb{E} V_{51}^{p}\left(\tilde{s}_p' | \mathbf{y}, \mathbf{s}\right) \\
\text{s.t.} \quad & c_p + c_c + a' = \Phi\left(\hat{y}_p\right) + (1-\tau) y_c + Ra \\
& a' \geq \underline{A}_{h_p} \geq 0
\end{aligned}
$$

where $\tilde{s}' = \left(a', y_p', y_c', s_p', s_c'\right)$.

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Government

- Runs balanced budget

$$G + SS + RB = B' + \tau \bar{Y}$$

- Set G so that $R - 1 = 4\%$ in steady state

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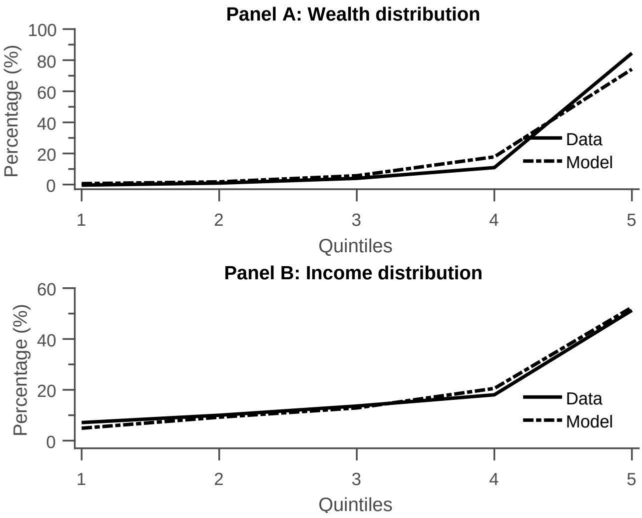
Wealth and income distribution



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